

number of them have potential for domestication and cultivation on commercial basis. Edible NWFPs received notable attention at the United Nations Conference on Environment and Development (UNCED) in Rio in 1992 and thereafter forest managers of several countries have recognized the importance to forest edibles in management of forests. However, there is conspicuous lack of understanding among resource managers and planners on sustainable harvest, value addition, equitable sharing of benefits, marketing, and conservation of these resources.

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## MENSURATION

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**Forest Measurements**

**Timber and Tree Measurements**

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**Yield Tables, Forecasting, Modeling and Simulation**

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### Forest Measurements

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### Introduction

The field of forest measurements is concerned with measurement, sampling, analyses, and prediction of properties and characteristics of the forest, including trees as well as other components of the forest ecosystem. In general, the main objective of forest measurement activities is to provide quantitative and qualitative data for forest management, policy-making or research. Forest measurements thus contribute substantially to the basis of forest-related human activities. Often, data acquisition is carried

out as part of a forest inventory, at estate, regional, or national level, or in scientific field experiments. Several forest measurement procedures can be used outside of the forest, for example in inventories of trees in urban streets and parks or in the landscape.

### Forest Measurements

Historically, science-based forest measurement procedures were developed mainly for aboveground parts of the trees and with the main objective of quantifying the wood resource and its growth potential. Including the planning, performance, and analyses of measurements of tree and stand attributes, this is usually referred to as the discipline of dendrometry (i.e., tree measurement), forest mensuration, or forest biometrics.

In the classical sense, measurements tend to be at the macroscopic level and are often carried out in the

field. Exceptions may be, for example, extracted wood cores that are taken to the laboratory for growth analysis, and the use of satellite, aerial, or other photographs for resource assessment. Carried out as part of a forest inventory and combined with land survey data, forest measurements provide information on individual tree properties as well as stand characteristics and overall forest structure for use in subsequent analyses and decision-making.

### The Contemporary Context

Due to an increasing diversity of the forest policy agenda and of forest management objectives the field of forest measurements is developing to provide data for a broader range of forest and site characteristics and to adjust and refine measurement procedures accordingly. This includes, for example, assessment of forest operations performance, regeneration abundance and quality, forest health, carbon stock, biodiversity (fauna, flora, and fungi), habitat diversity, range land, water resources, light conditions, crop nutrient balance, soil characteristics, recreation opportunities, and cultural, heritage, and amenity values.

For many of these topics, sampling and measurement procedures have been developed in other branches of science. In the context of forest ecosystem management, the challenge is to combine these procedures cost-effectively with forest measurement practices. Many of the additional ecosystem attributes correlate well with individual tree properties, stand characteristics, or overall forest structure, and traditional forest measurements are often more cost-effective and versatile.

So, regardless of management objectives, the trees of the forest must be quantified for informed decision-making. Consequently, forest measurements maintain a strong focus on the aboveground tree components of the forest ecosystem.

### Measurement, Sampling, Analyses, and Prediction

Forest measurement activities generally comprise direct and indirect measurement, sampling, analyses, and prediction. Obviously, mathematics, statistics, and computer science are fundamental to forest measurements, and several measurement techniques are borrowed or adapted from engineering, land surveying, photogrammetry, and other professions.

Due to the forest environment and the size of trees special instruments have been developed for field use. These include calipers to measure tree diameter, relascopes to measure stand basal area, hypsometers to measure tree height, and xylometers to measure wood volume.

In general, due to the huge number of trees present on most forest land, only a sample is measured. Sample values are then expanded appropriately to obtain estimates for the population of interest. General sampling theory provides the foundation, but several sampling techniques have been developed specifically to forest conditions.

Except for the purpose of timber trading, forest measurements are most often based on nondestructive sampling. Consequently, the prediction of quantities, qualities, and events other than those directly measured plays a major role for utilizing the potential of forest measurements. The prediction of stemwood volume is probably the most notable example. A more complex but equally important example is the prediction of forest dynamics in response to management actions. As a general recommendation, models should consider the simultaneous nature of state variables and account for relevant interactions as well as temporal and spatial correlations in data.

### Overview

A comprehensive review of all methods used for forest measurements is beyond the scope of this contribution. Here, the focus is on general principles, definitions, variables, instruments, methods, and models that are widely used for the assessment of some basic tree properties and stand characteristics.

Following an introduction to general measurement definitions and principles and a summary of the history of forest measurements, the scope of dendrometry is outlined and forest measurement practices explained for variables age, stem number, diameter, girth and basal area, bark, tree and stand height, stem taper, form factor, and wood volume. More specific approaches and details, including remote sensing measurement techniques and forest modeling based on dendrometric data, are covered elsewhere in this Encyclopedia (*see Resource Assessment: GIS and Remote Sensing, Mensuration: Yield Tables, Forecasting, Modeling and Simulation*), in forest measurement and modeling textbooks, and in similar texts from other professions and sciences.

### General Measurement Definitions and Principles

By facilitating comparison across time and space, consistent and objective measurement principles contribute to an unambiguous interpretation of observations which, in turn, may lead to an extension of human experience and knowledge. Measurements thus contribute to a huge variety of human activities and interactions.

### Definition, Scales, and Units

In its broadest sense, measurement can be defined as the rule-based assignment of numerals to physical objects and natural phenomena. Numerical quantities can be assigned under different rules using different kinds of scales and different kinds of measurement procedures. Any scale of measurement may be classified as being a nominal, an ordinal, an interval, or a ratio scale.

A nominal scale is used for numbering or counting objects or phenomena of a certain identity (for example, number of live trees). An ordinal scale is used to express rank or position in a series (for example, numerical codes 0, 1, 2, 3, or 4 for tree sociological classes dead, suppressed, intermediate, codominant, or dominant, respectively). An interval scale includes a series of graduations marked off at uniform intervals from an arbitrary origin (for example, temperature). A ratio scale is similar to an interval scale, but implies an absolute zero of origin (for example, stem diameter, tree height, and wood cubic volume). Ratio scales are the ones most commonly applied in forest measurements.

The analysis and interpretation of observations must take into account the measurement scale. For example, the number and type of legitimate mathematical operations depend on measurement scale. Although different measurement units are used in different parts of the world, numbers generally follow the decimal system, while the SI system offers comparative advantages over possible local units.

### Variation, Accuracy, and Precision

Variation caused by uncontrolled factors, both known and unknown, is called error. The main sources of variation include properties intrinsic to the measurement object as well as external factors in sampling, measurement, and analysis of data.

Any measurement is subject to error or, in other words, deviation from a true value which generally remains unknown. It is often useful to identify, estimate, and reduce this source of variation relative to other sources. Errors include systematic and random components and originate in measurements due to measurement object, instrument, procedure, or the imperfection of human senses.

Systematic errors are constant or functionally dependent on their cause. These should be identified and eliminated as far as possible. Random errors are normally and independently distributed with zero mean and common variance. They are due to several possible causes, none of which dominates the measurement process. Random errors account for unexplained variation and are inherent to any measurement.

For any application of measurements, results are only as reliable as the input. So, other things being equal, the measurement error should be minimized for increased accuracy and precision. Accuracy is a measure of reliability as indicated by the difference between the true value and the most probable value derived from a series of measurements. Bias is defined conversely of accuracy. Precision is a measure of repeatability and is the degree of agreement between individual measurements in a series of measurements of the same quantity. In practice, accuracy is often judged from precision although this may lead to misinterpretation.

### History of Forest Measurements

Assessment of forest resources, including the production and harvest potentials, has probably been crucial for exploitation by people and land managers ever since trading of natural resource products began and has definitely played a vital role during history for central authorities of societies that depended on forest products. Although timber is useful or even a necessity for the construction of houses, fortifications, bridges, carriages, and ships, the forest also provides firewood, fencing material, berries, fodder, grazing, and a host of other commodities.

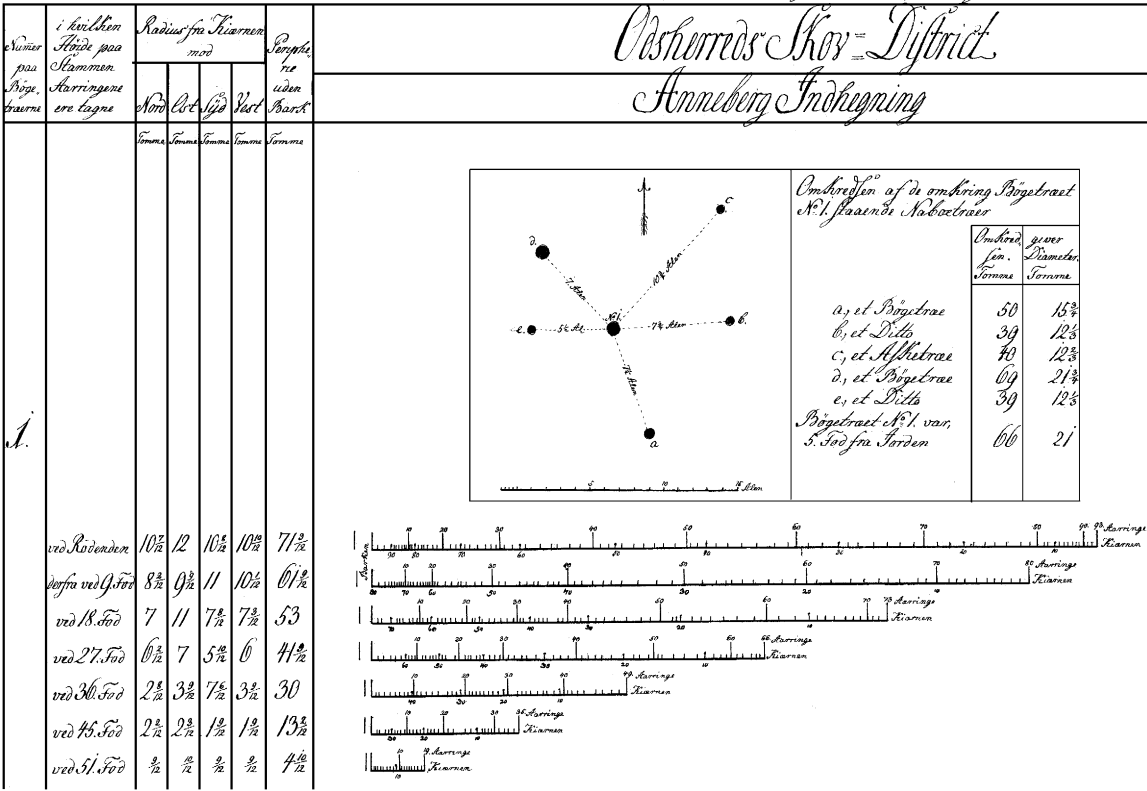
Scarcity of resources obviously provides an incentive to initiate forest measurements. It is known that early societies in many parts of the world developed local forest measurement practices and rules to ensure reliable estimates of forest resources. Some of these even took approaches that are very similar to modern scientific methods and principles.

### The Scientific Approach

The scientific approach to forest measurements in a modern context began in Central Europe at the introduction of regular and planned forestry in the mid-1700s, coinciding with the advent of forest science and modern natural sciences in general. Early initiatives concentrated on the identification of key variables for estimation of wood volume and prediction of growth, and on measurement procedures (Figure 1).

The establishment of the German Federation of Forest Experiment Stations in 1872 furthered a major breakthrough which resulted in a common norm for tree and stand variables in Central Europe. Following the foundation of the International Union of Forest Research Organizations (IUFRO) in 1892 these recommendations have greatly influenced forest measurement developments and practices in other parts of the world. However, no internationally agreed standards have emerged (Figure 2), but there

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Odsherreds Skovs Distrikt  
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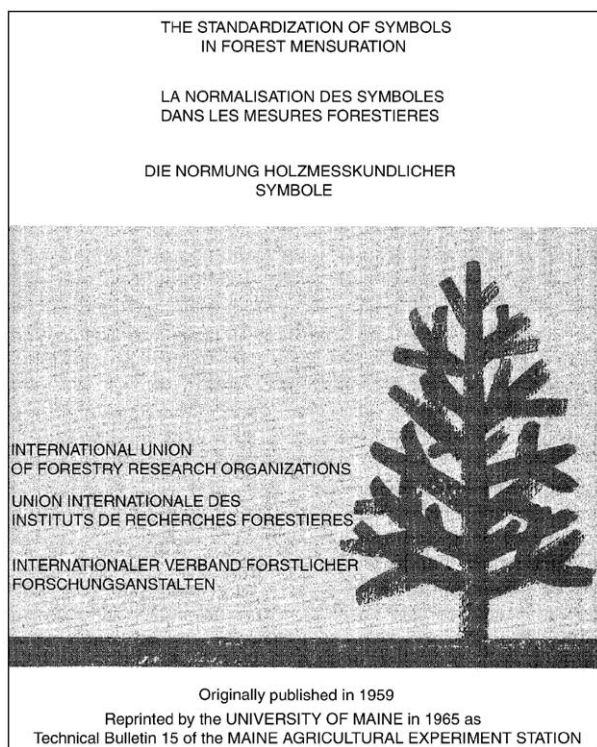
**Figure 1** An example of early forest measurements to a modern scientific standard. In 1793, C.D.F. Reventlow initiated dendrometric measurements in stands of oak (*Quercus*) and beech (*Fagus*) in Denmark. Analyzing the size and past growth of trees in relation to the size of neighboring trees, he constructed accurate yield tables and prescribed economically optimal thinning practices that still prevail. Original record sheet for beech tree no. 1 in Anneberg Fenced Forest, Odsherred District. The sketch is a stem map for tree no. 1 and its closest neighbor trees A, B, C, D, and E, recording location, species, and circumference at the butt for each tree. Below to the left, measurements of stem radius (towards north, east, south, and west) and circumference of tree no. 1 at the butt and up the stem at regular intervals. To the right, measurements of annual rings at the same locations in the stem, indicating ring width and age. Number of years is counted from the center (right) as well as from the bark (left). From the archives of the Danish Museum of Hunting and Forestry/Society of Forest History.

appears to be a general consensus on some basic principles and variables. The most notable and universal key variable is diameter (or, alternatively, basal area) at 1.30 metres above ground level, often referred to as diameter at breast height (dbh). Several of the historical forest measurement variables still prevail, although they were conceived during a period when the science of statistics was only emerging and calculations had to be carried out manually or using a slide rule rather than electronic data processing. Also, market demands for more uniform raw material due to the industrialization promoted a strong focus on timber production. An analytical, more holistic approach to classical dendrometry, unbiased by traditions of the profession and considering the range of issues in forest ecosystem management, would probably result in more comprehensive and statistically rigorous choices and definitions of variables and methods.

**Dendrometry**

The dendrometric components of a tree depend on measurement objectives and local traditions, but generally include stem, branches, foliage, bark, stump, and roots (Figure 3). Each of these may be measured with or without bark, individually or together, split in parts, or taken as a whole. Next, measurements may be carried out for standing trees, dead or alive, or for felled trees or wood products. Supplemented with area-based measurements, for example of sample plot area and stem number, measurement values for individual trees are expanded to obtain estimates at the population or stand level. Instruments for measuring or estimating dimensions of trees or forest products are collectively referred to as dendrometers.

Dendrometric measurements concentrate mainly on wood volume or, more precisely, measurements of

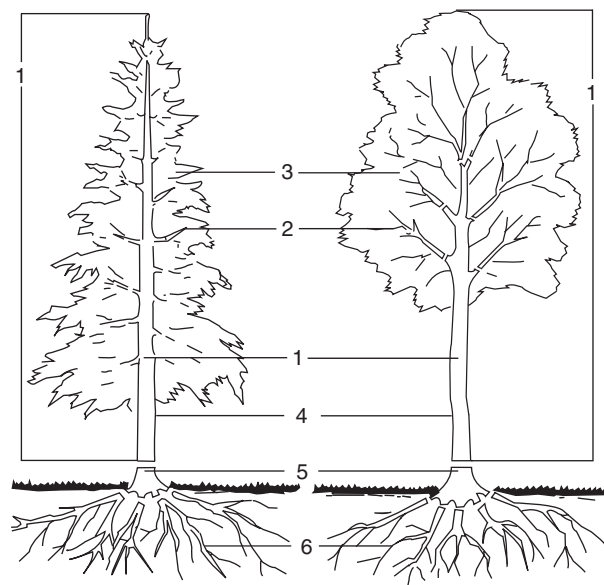


**Figure 2** Forest measurement standards. Although seminal initiatives have been taken in Central Europe already in the late 1800s, no internationally agreed standards for forest measurements have emerged. Following approval from member organizations in 1956 IUFRO launched recommendations on the definition of variables and the standardization of symbols in forest mensuration. Although these recommendations were published simultaneously in English, German, and French, their use varies considerably between continents and countries.

tree and stand attributes from which wood volume can be derived. For commercial purposes the main concern is merchantable volume of the main product, often one or more parts of the stem. Correspondingly, merchantable height (at which stem diameter or exterior wood quality is at its merchantable limit) is assessed directly in the field or derived from models of stem taper or total stem volume.

Due to different growth habits different tree species or species groups generally differ significantly in their dendrometric characteristics. This also holds for age, site, and stand treatment effects. Consequently, the identification of tree species or species group is often needed to capture and model the variation in tree sizes and shapes. However, in most situations, the volume of harvested products can often be assessed using general models that are independent of species or species group.

The most frequently used forest measurement variables include age, stem number, diameter, girth, basal area, bark thickness, height, stem taper, form factor, and volume at tree and stand level. In the



**Figure 3** Dendrometric components of a tree: 1, stem; 2, branches; 3, foliage; 4, bark; 5, stump; and 6, roots. Depending on measurement objective and local traditions, measurements may exclude undersized material (size limit usually defined in terms of stem, branch, or root diameter), they may exclude or include bark, or bark may be measured separately. Generally, foliage is not considered except for scientific purposes, for health assessment, or in the case of whole-tree harvesting.

following text, symbols for these and other dendrometric variables generally follow recommendations from IUFRO (Figure 2). By convention, uppercase symbols are used to denote stand values, whereas lowercase symbols refer to individual trees.

## Age

For many practical and scientific purposes chronological time and age are considered fundamental variables for understanding and predicting the dynamics of tree and stand growth and other processes and events in the forest.

## Definitions

By definition, the chronological age,  $t$ , of an individual tree is calculated from the time of germination of the seed or from the time of budding if the tree originates from a cutting or a sprout. Alternatively, age in a forestry context may be calculated from the time of planting or from the year in which the tree reached a certain reference height, for example 1.30 m above ground level. The latter is often preferred if early growth is hampered by browsing or severe weather such as drought or late frost. For coppiced trees, the age of stems may differ from the age of stools and roots.

In the case of strictly even-aged forest stands where all trees germinated, rooted or sprouted in the same year, stand age,  $T$ , is either known from management records or can be determined quite accurately from sampling. Otherwise, if a stand appears even-aged and the age of individual trees ranges within limits, stand age generally refers to some sample average. In the case of uneven-aged forest stands, age is hardly a main concern and is often disregarded.

Considering the longevity of trees, the scale of resolution for age is often 1 year, but any measure of time may be used.

### Measurement Principles and Instruments

With distinct annual rings age is easily determined from counting on stumps or radial wood cores. Radial wood cores are extracted using an increment borer, a hollow auger that is screwed into the stem to remove a thin cylinder of wood. The main sources of error include incomplete, partial, and false growth rings, incorrect adjustment for age at the point of sampling, and failure to include the pith in core samples. The most reliable ring counts are made on complete cross-sections.

For some tree species, age may be determined alternatively from counting of branch whorls. In this case, errors may originate due to broken tops. Another important variation of the theme is dating of stumps to determine time and harvest volume of past cuts.

In the absence of seasonal growth rings and other reliable age indicators, age may be derived from observed correlation with other variables of the forest ecosystem. This method is subject to large variation.

### Applications

Age determination of trees and stands is essential for even-aged forest management where age or, alternatively, a given period of observation, is often used in the prediction of growth, for site classification and for calculations leading to an optimization of economic return. For uneven-aged forest management age generally remains an inferior variable.

### Stem Number

One of the most simple characteristics of a forest stand is the number of trees present, either dead or – more commonly – alive.

### Definitions

In a forestry context, stem number is a more operational variable and frequently replaces the

number of live trees. Generally, stem number is expressed in count per unit area. Sometimes, stem number refers only to trees above a certain size.

### Measurement Principles and Instruments

Stem number at stand level,  $N$ , may be derived from stem counts,  $n$ , on sample plots or estimated based on counts at representative points in the stand.

For a plot,  $i$ , of known area,  $a_i$ ,  $N_i = n_i/a_i$ . In the case of multiple plots per stand, the stand average may be weighted by plot area or another relevant factor. The number of plots needed to achieve a reliable stand level estimate depends mainly on the required precision, the stem number level, and its variation across the stand. Obviously, the error in determination of area propagates to stem number at stand level. When trees occur in rows spaced at regular intervals, counting may be carried out in a sample of these, and stem number at stand level derived from row length and sample fraction.

In dense stands, stem number can be pragmatically judged using a string of fixed length. Tied to a randomly located tree, the length of the string equals the radius of a circle of known area. For example, if the string is 3.99 m long, stem number per hectare equals stem count times 20. An alternative, pragmatic method is to measure the distance,  $k$ , from a sample point,  $i$ , to the  $n$ th nearest neighboring tree. Then,  $N_i = 10^4(n - 1/2)/(\pi k_i^2)$ . The efficiency and choice of these techniques depend on the pattern of variation in the stand and the ease of measurement.

### Applications

Combined with other variables such as age or size of the trees, stem number provides an immediate impression of stand density for a given species or forest type. Consequently, stem number is widely used in forestry, for example in thinning decisions or in quality assurance of thinning operations. Stem number is also a significant variable in volumetric calculations.

### Diameter, Girth, and Basal Area

Diameter, girth, or cross-sectional area of the stem or other woody parts of the tree is frequently used as an indicator of size, to calculate wood volume, or as a predictor of other tree and stand properties. This text mainly refers to measurement of the stem of standing trees, but similar principles apply to any other measurement of diameter, girth, and cross-sectional area.

## Definitions

For standing trees, the most widely used tree and stand characteristic is stem diameter,  $d$ , measured outside of the bark at breast height, often referred to as diameter at breast height (dbh). In countries that use the SI system breast height is generally located at 1.30 m above ground level. Alternatively, some other reference height may be used, for example 4 feet 6 inches.

Assuming that stems are circular, stem cross-sectional area or, in the forestry terminology, basal area,  $g$ , is calculated as  $g = d^2(\pi/4)$ . Alternatively, diameter and basal area may be derived from girth or stem circumference,  $c$ , as  $d = c/\pi$  and  $g = c^2/(4\pi)$ .

At the tree level, stem diameter, girth, and basal area can be used interchangeably, but generally diameter is the preferred variable because it is easier to visualize. Stand basal area,  $G$ , i.e., total basal area per unit area of land, is a specific characteristic of stand density.

Obviously, the stem and other woody parts of a tree are generally not exactly circular in cross-section. Consequently, the objective of any tree diameter measurement is to obtain the diameter of a circle with the same cross-sectional area as the measured part of the tree. The deviation from the circle depends on tree species, terrain, and wind conditions. In the case of substantial and consistent deviations a model to adjust measurements may help provide reliable estimates of basal area.

Another problem is the universal reference level at breast height. This provides an apparently consistent standard and a convenient measurement height, but is biologically not well justified. In practice it is difficult or impossible to determine the exact location of ground level due to natural variations at the base of the tree, and for the individual tree ground level may change over time, for example due to subsidence, erosion, or changes in the amount of organic material. However, definitions such as the highest, lowest, or average point of the ground surface touching the tree are usually considered sufficiently accurate.

Depending on tree species, forest type, terrain, weather, and other conditions a number of irregularities may occur. The most common problems include stem swell, wound callus, forking, lean, and buttressing. Local stem irregularities may be circumvented by measuring above or on either side of the irregularity, and when trees fork below breast height each fork is usually considered an individual stem. Severe and frequent buttressing usually requires an alternative reference level above breast height.

For small trees, and especially those below breast height, another arbitrary reference may be used, for

example 10 cm or a level consistent with measurement points for stem taper or form factor. However, measuring the diameter of trees shorter than breast height is generally only considered for scientific purposes.

## Measurement Principles and Instruments

The diameter of stems, branches, stumps, and roots is most frequently measured directly using a caliper (Figure 4) or indirectly using a tape graduated in  $\pi$  units. Usually 100 cm is the size limit for caliper measurements. Reliable electronic calipers are now available and are widely used.

As an alternative to caliper and tape, indirect measurement may be carried out using a fork or a stick with two arms or sighting lines forming tangents to the stem and a scale graduated according to this geometry. A similar principle is used for remote, optical measurement of diameter. For scientific purposes, diameter and diameter change may be monitored on a continuous basis using a band dendrometer (periodic readings) or a dendrograph (continuous record) encircling the stem or a branch.

In the absence of previous measurements and when growth occurs in a traceable, seasonal, or annual pattern information on past dimensions and growth performance can be obtained from the analyses of growth rings, for example from extracted wood cores, cut trees or stumps (see section 'Age' above). This approach is sometimes used for inventory purposes, in scientific investigations, and for dendrochronological dating (see **Mensuration: Tree-Ring Analysis**). A main concern with this method is the representativeness of samples.

Errors in determination of diameter, girth, and basal area are mainly due to measurement object (the tree), instrument, class grouping, or observer. In general, the relative error is small compared to other forest measurements.

Obviously, diameter, girth, and basal area change throughout the growing season and for that reason time of measurement should be chosen consistently and considered in subsequent data analysis. Even in the case of a distinct growing season size during the rest period may fluctuate depending on weather conditions, but generally this does not produce error of any significant magnitude.

For convex as well as concave cross-sections, a caliper diameter is the directly measured distance between parallel tangents to the region of a convex closure, whereas a taped diameter is derived from a parametric measurement of the convex closure. By definition, the diameter of the measured cross-section should equal the average of diameters measured over all possible directions. Consequently,





**Figure 4** The caliper is the most common instrument in forest measurements. It is used for direct measurement of diameter of stems, branches, stumps, and roots. The conventional forestry caliper consists of a graduated bar with two parallel arms at right angles; one arm is fixed on the bar, while the other slides on it. Calipers made of metal or carbon fiber are more reliable than those made of wood, because they are sturdy, weather-proof, and easy to clean for resin and dirt. Modern calipers operate electronically with data storage directly in the caliper or data transfer to a portable field computer.

taped measurements are more precise, but result in a slight overestimation of diameter and basal area. In contrast, the caliper is theoretically unbiased for direct distance measurement and the average error may be reduced through the use of random measurement directions.

Instrument error due to a loose caliper arm is likely to result in underestimation, whereas error due to natural fluctuations in the length of a tape is randomly distributed. Each of these can be reduced to a minimum through the use of properly manufactured and well-maintained equipment. For optical instruments the use of a calibrated prism, a magnifier, a split-image, and a tripod improves precision considerably. Generally, alternative instruments are less precise than caliper and tape.

When using an analog instrument diameter is often recorded to size classes. The error due to class grouping depends mainly on class width and the distribution of tree sizes. Except for very thin trees, the error for 1 cm classes is negligible.

Errors due to the observer include failure to identify breast height correctly or consistently, failure to measure perpendicular to the length axis of the tree, and varying or inconsistent contact pressure.

The error due to incorrect measurement height may be one- or two-sided and depends mainly on stem taper. In the case of heavy snow cover, floodwater conditions, or similar irregularities, a stick may be used as a probe to locate true ground level. Tilting always results in overestimation of diameter and girth. The problem with contact pressure depends on bark characteristics. Special care should be taken for species with soft, scaly, or otherwise irregular bark.

If successive measurements are taken on the same tree, for example on continuous inventory plots or in scientific experiments, accuracy can be improved considerably by permanently marking the exact location of the breast height reference level. If a caliper is used, accuracy can be further improved by taking two perpendicular measurements and measuring in the same two directions on each measurement occasion ( $d_1$  and  $d_2$ ). Typically, basal area is then calculated based on the quadratic, arithmetic, or geometric mean diameter, where  $\sqrt{(d_1^2 + d_2^2)/2} \geq (d_1 + d_2)/2 \geq \sqrt{d_1 \cdot d_2}$ .

As a rule-of-thumb for trees with a “regular” cross-section, individual tree diameter can be measured to within  $\pm 2.5\%$  of the true value when using a tape or two perpendicular caliper measurements. Often,



diameter is recorded to the nearest 1 cm in forestry practice and to the nearest 1 mm in high-precision scientific investigations. As a rule of thumb stand basal area can be measured to within 1–2% of the true value.

As an alternative to diameter and girth measurements, stand basal area may be estimated directly using a relascope for angle count sampling. The relascope is used to discriminate between trees depending on whether or not the tree subtends an angle equal to or greater than that of the relascope gauge when viewed from the sampling point (Figure 5a). Counting from a random point the stems whose diameter at breast height exceeds a certain angle produces an estimate of stand basal area (Figure 5b).

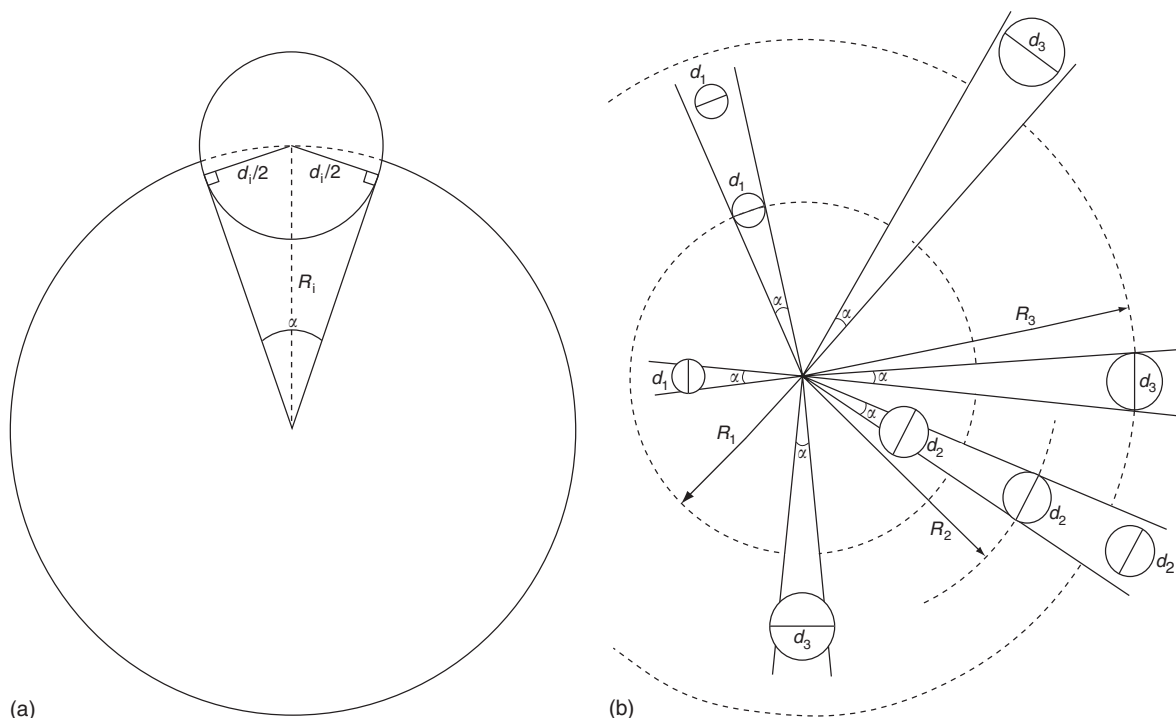
A stick equipped with a notched metal plate at one end is an example of a simple relascope. More sophisticated versions include, in order of increasing precision, the wedge prism, the optical mirror relascope, and the telerelascope. In the absence of an instrument, the observer's thumb can be used to obtain a crude estimate of basal area, provided

proper 'calibration' at an arm's length to a personal count factor.

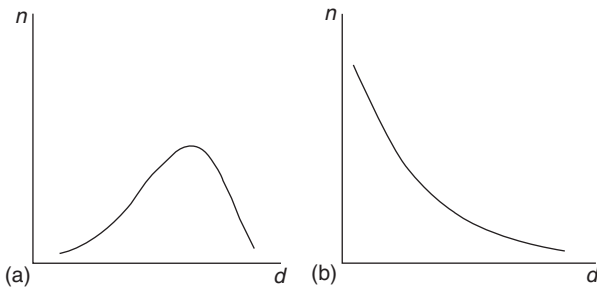
### Applications

Diameter at breast height provides an immediate impression of the measured object and is the most widely used general characteristic of standing trees and forest ecosystems. Often, diameter at breast height is measured for all trees of a sample, and several other tree and stand characteristics are then modeled based on this quantity. Derived properties commonly include height, stem form, wood volume, volume growth, and branch size. Models for each of these may be improved by including additional variables. For example, upper stem diameters for estimation of stem form and volume.

Full enumeration of individual tree diameters provides substantial information on the sociological structure of a forest stand, including the size class distribution (Figure 6). Commonly used stand variables includes arithmetic mean diameter,  $\bar{D}$ , and quadratic mean diameter,  $D_g$ . While the arithmetic



**Figure 5** Angle count or point sampling. (a) Given a random point in a stand of trees of varying diameters  $d$  such that a particular tree of diameter  $d_i$  exactly subtends the relascope angle  $\alpha$  when viewed from the random sampling point, the center of this tree is the distance  $R_i$  away from the sampling point, where  $R_i = d_i / [2 \cdot \sin(\alpha/2)]$ . The radius  $R_i$  describes a circle or sweep of area  $\pi R_i^2$ . (b) All trees of diameter  $d_i$  within this circle subtend an angle greater than or equal to  $\alpha$ . The ratio of the basal area of these trees to the area of the circle equals  $\sin^2(\alpha/2)$  independently of  $d_i$ . Consequently, a count  $n$  of all trees subtending an angle greater than or equal to  $\alpha$  provides an estimate of the sum of basal area of these trees within each of their circles, i.e.,  $Gha^{-1} = n \cdot [10^4 \sin(\alpha/2)]$ , where  $[10^4 \sin(\alpha/2)]$  is called the count or basal area factor. Borderline trees exactly subtending the angle should be counted as  $\frac{1}{2}$ . For the example in Figure 5b  $n$  equals  $4\frac{1}{2}$ . Count accuracy is considerably improved by choosing an appropriate count factor, using steady instrument support, checking for hidden trees, inspecting the size of leaning and borderline trees, adjusting for slope in line of sight, and avoiding stand edges.



**Figure 6** Smoothed diameter distributions (stem number,  $n$ , vs. diameter,  $d$ ) for two different forest types. (a) An even-aged monospecific stand, and (b) an uneven-aged mixed species forest. The size class distribution is widely used in analyses of the sociological structure and dynamics of tree populations, and is a powerful tool for scenario modeling and decision-making in forest management. Commonly used models include continuous distribution functions as well as discrete functions and matrix models. Models may be univariate, as in this example, or multivariate, taking into account the simultaneous distribution of, for example, diameter and height of individual trees. For prediction and projection purposes parameter values are often recovered or predicted from relatively simple mensurational tree and stand characteristics.

mean relates immediately to the size class distribution, the quadratic mean is equal to the diameter corresponding to mean basal area. In forestry terms, the quadratic mean is often considered more meaningful because of its direct relation to basal area and, in turn, stand volume. Diameter enters quadratically into volume calculations, which are often based on either size class distribution or mean tree dimensions.

Stand basal area is an essential variable for forest management, for example for thinning decisions. In managed forests, individual tree diameter is strongly influenced by thinning practices, and stand mean values thus directly reflect management practices and their effects on growth. For a given forest type management guidelines are often based on an index or decision criterion that combines stand basal area with stand height, stem number (live trees), or mean tree size.

In addition to dendrometric tree and stand properties, diameter also serves as an input variable for harvesting criteria, assortment distributions, stumpage prices, forest valuation, and a range of other uses. Ultimately, through a number of model links, diameter at breast height is *the* key variable for decision-making in forest management, including ecological as well as economic and social aspects.

## Bark

Measurement of bark thickness or bark volume may be needed to quantify bark for harvesting or to convert from over bark to under bark dimensions.

## Definitions

Mensurationally, the bark comprises all tissues outside of the xylem, including the cambium. Bark thickness is generally defined as the difference in radius of two concentric circles, one defined by the bark surface, the other by the interior wood surface.

## Measurement Principles and Instruments

On felled trees bark thickness can be measured at the cut ends or on bark samples that are cut off. On standing trees bark thickness may be measured using a probe or a gauge with a nail or a blade that is pushed through the bark, perpendicular to the stem periphery, to meet the wood surface. The depth of penetration is measured. Alternatively, bark thickness may be measured on extracted cores (see section 'Age' above) or on small pieces of bark that are cut off. Often, two or four measurements are made at diagonal points, and often at the points where the tree was calipered.

Measurements of bark thickness are often subject to large variation mainly due to irregular bark, difficulties in penetrating the probe or gauge exactly to the wood surface, slanted boring, and compression of the bark when using the instrument (probe, gauge, or increment borer). In the case of a distinct seasonal growth pattern, measurement accuracy usually deteriorates during the growing season.

When bark is a main commercial product more accurate methods may be needed. This normally implies cutting out samples of bark, for example squares of  $20\text{ cm} \times 20\text{ cm}$ , to determine thickness, volume, or weight. These may also be used for assessment of bark quality. The size and distribution of samples should be optimized based on statistical correlation with harvest quantities.

Bark volume may be derived from thickness or measured directly using a xylometer. Xylometry is also used for direct measurement of wood volume (see section 'Volume' below). The bark (or another object) is submerged into water to determine water displacement or buoyancy. If the bark is highly irregular, sample volumes measured by xylometry may underestimate stacked or piled volume.

The weight of bark is easily measured directly. Due to large variation within and between trees and seasonal fluctuations in moisture content, objective measurements require that the bark has been dried to a specified level. For operational purposes, the bark may be allowed to dry in open air for a given period to achieve equilibrium moisture content before being measured.

## Applications

Bark thickness varies with tree species, genotype, site conditions, age, stand treatment, tree size, growth rate, health, height above ground, and geographic orientation. For many species, the average volume of bark ranges from 5 to 20% of the over bark volume.

When trees are measured over bark, bark measurements may be needed to estimate diameter, basal area, or volume under bark. This is essential mainly when sale of commercial timber is based on under bark dimensions, or when growth estimates are based on extracted wood cores or stem cross cuts. In the latter case, bark growth should also be considered. Often, bark volume or under bark wood volume is calculated based on the simple model  $b_v = ((d_{ob}^2 - d_{ub}^2)/d_{ob}^2) \cdot v_{ob}$ , where  $b_v$  is bark volume,  $d_{ob}$  is stem diameter over bark,  $d_{ub}$  is stem diameter under bark, and  $v_{ob}$  is volume over bark.

When bark is a main product, requirements on accuracy of measurements and models obviously relate directly to the commercial value of the bark. Bark is often traded by weight, which may be measured directly or predicted from other dendrometric variables.

## Height

The height of individual trees as well as stand height are widely used in volume calculations, in the estimation of site productivity, and for a range of other objectives. Height measurement is more demanding and less precise than measuring the length of felled trees or wood products.

## Definitions

The height of a standing tree,  $h$ , may be defined in different ways, depending on usage and measurement traditions. For total height, two slightly different definitions prevail: the shortest distance either between base and top of the tree, or between ground level and top of the tree. These definitions result in different measures of height only for leaning trees. The first definition may be preferred because it is often compatible with the length of felled trees and a consistent measure for volume calculations, whereas the second definition may be generally more practical. Other height definitions may refer, for example, to merchantable parts of the tree.

Another problem with definition of tree height is the exact location of ground level and of the base and the top of the tree. The principles for location of ground level and tree base is similar to those used to define the location of breast height (see section 'Diameter, girth, and basal area' above). Except for

trees with a very distinct tip, it can be difficult to locate the top. This may be due to deliquescent growth, drooping branches, an irregular crown, or crown dieback. Consequently, the top of a standing tree is often defined as its highest growing point.

## Measurement Principles and Instruments

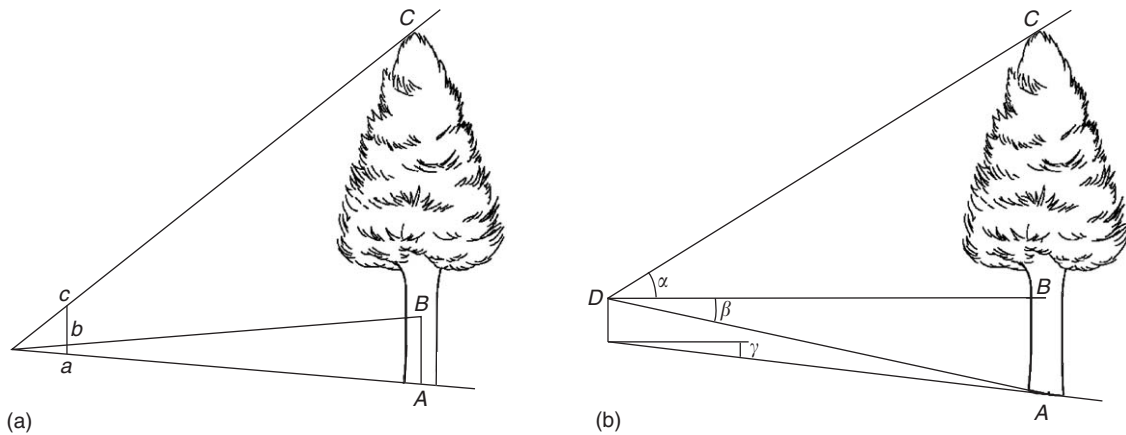
Tree height and any other measure of height along the stem may be determined by direct or indirect measurement. Instruments for measuring the height of standing trees are collectively referred to as hypsometers. Several such instruments have been developed specifically for use in the forest.

Due to poor sighting conditions and the time needed for instrument handling it is generally slow work to measure tree height and a much smaller sample is measured than for diameter or basal area. Sampling principles and sample size should be determined based on requirements for end-use precision balanced with measurement costs. As an alternative to measuring standing trees, height estimates may be obtained from the length of felled trees that are sampled for this purpose.

Depending on tree size and stand density direct height measurement may be carried out using a graduated rule, stick, or pole. Extendable poles may be equipped with an engraved or colored scale on the outside or a self-winding tape measure in the inside. Generally, direct measurement is practical only for trees shorter than 10–15 m, and often indirect measurement is used even for these.

Indirect height measurement is based on geometric or trigonometric principles (Figure 7). The geometric principle relies on the use of similar triangles, whereas the trigonometric principle makes use of trigonometric functions. In both cases, the accuracy and precision of hypsometers generally depend more on construction details and fine mechanics than on measurement principle. However, with the advent of electronic hypsometers based on high-precision angle measurement combined with acoustic or laser technology for distance measurement, the tendency is that trigonometric hypsometers prevail. This technology has increased reliability of height measurements considerably, but not to a standard similar to that required in land surveying.

In the absence of previous measurements and when height growth occurs in a traceable, seasonal, or annual pattern, information on past height and growth performance can be obtained from measuring back to the terminal bud in previous years (see section 'Age' above). For species without annual whorls, reasonable estimates of height growth can be obtained from counting growth rings at intervals along the stem. If diameter growth is also analyzed



**Figure 7** Indirect height measurement based on geometric or trigonometric principles. (a) The geometric principle relies on the use of similar triangles. In this example, a vertical ruler on the hypsometer is adjusted to fit the reference measure  $|ab|$  with the length  $|AB|$  of a pole at the base of the tree. Next, total tree height  $|AC|$  is read directly off a scale which is graduated according to the known proportion between  $|ab|$  and  $|AB|$ , i.e.,  $|AC| = |ac| \cdot |ab|/|AB|$ . (b) The trigonometric principle relies on the use of trigonometric functions. In this example, the angles  $\alpha$  and  $\beta$  are measured with the hypsometer, while the horizontal distance  $|BD|$  between tree and observer is determined independently. Total tree height  $|AC|$  equals  $|BD| \cdot (\tan \alpha + \tan \beta)$ . If horizontal distance is not measured directly the distance between tree and observer needs adjustment for terrain slope  $\gamma$  by a factor  $\cos \gamma$ . The angle scale of the hypsometer may be replaced by a nonequidistant tangent or percentage scale, so that tree height is easily calculated directly from hypsometer readings.

this may provide a complete record of past growth. This method is known as stem analysis.

Errors in height measurement are mainly due to measurement object (the tree), instrument, or observer. The most common field problems include leaning trees, parallax error in sighting, and incorrect identification of the tree top. While the latter can be avoided by careful observation, the effect of these problems is essentially similar: sighting is not done to a point located vertically above the geometric center of the base of the tree.

Generally, only a small fraction of trees is measured for height, so trees that lean severely can be avoided. Otherwise, simple geometry and trigonometry can help provide an estimate of tree height that is close to the true value. When parallax error is not due to hypsometer construction this can be avoided by careful observation.

Instrument errors originate in the construction or calibration of hypsometers. Whereas bias in analog hypsometers is often one-sided, more subtle but significant patterns may originate with electronic hypsometers. Outdoor work conditions and the sensitivity of mechanical and electronic components is a critical combination that stresses the need for regular test and calibration.

The random error in height measurement depends highly on tree species, forest type, hypsometer quality, and whether instrument support is used. As a rule of thumb individual tree height can often be measured to within  $\pm 2.5\%$  of true height, but measurement errors of up to  $\pm 5\%$  or more may

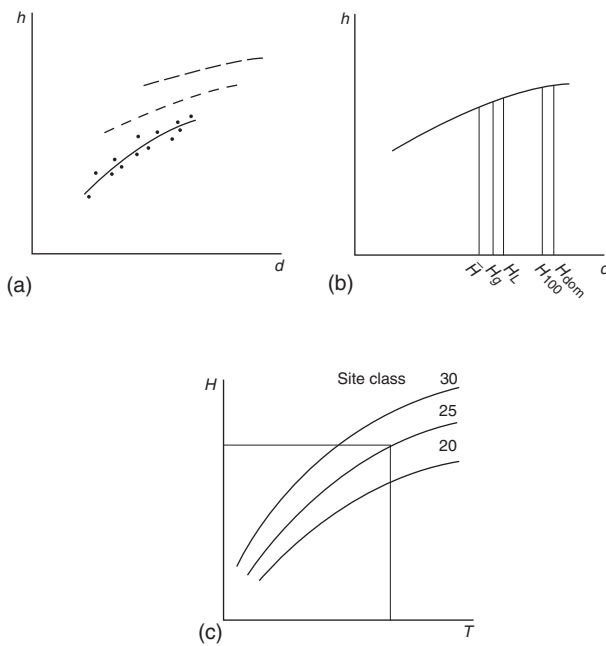
occur. Electronic hypsometers and the use of instrument support may help increase precision. Often, individual tree height is recorded to the nearest 10 cm.

### Applications

Tree and stand height as well as their distribution and growth provide essential information on forest and stand conditions and are used for a range of different objectives. For example, height is used in the estimation of wood volume, volume growth, stem form, and site and stand productivity, and it is incorporated in more complex models of forest dynamics. Height is also used more directly for management decisions, for example as a criterion for thinning of conifer plantations at a high risk of windthrow.

The potential uses of tree height depend highly on forest type. For even-aged forest individual tree height is often modeled as a static, age and stand specific function of diameter at breast height (Figure 8a). For uneven-aged forest interpretation of the relationship between tree height and diameter is less straightforward due to a more complex stand structure, and mathematically more flexible models are needed.

Different indicators of stand height may be derived from the model or directly from sample values. Commonly used indicators of stand height include arithmetic mean height,  $\bar{H}$ , height corresponding to mean basal area,  $H_g$ , mean height weighted by basal



**Figure 8** Tree and stand height in an even-aged, monospecific forest. (a) Individual tree height,  $h$ , as a function of diameter,  $d$ , at breast height (full line). If remeasurements take place, for example as for continuous forest inventory plots or scientific experiments, a family of curves (dashed line) may be derived that reflects site characteristic height growth for a given stand treatment. In this example there are three remeasurements, with individual observations (•) included only for the first one. (b) The relative position of different indicators of stand height ( $\bar{H}$ ,  $H_g$ ,  $H_L$ ,  $H_{100}$ ,  $H_{dom}$ ). For readability, labels for stand height appear on the  $d$ -axis. (c) Classification of site and stand productivity by age,  $T$ , and stand height,  $H$ . In this example, the typical height–age development has been shown for three site classes (stand height 20, 25, and 30, respectively, at a given reference age), and a stand has been classified.

area,  $H_L$ , top height or average height of 100 thickest trees per hectare,  $H_{100}$ , and dominant height or average height of sociologically dominant trees,  $H_{dom}$  (Figure 8b). Historically based on arguments of computational ease rather than statistical rigor, each of these was conceived for specific purposes, generally aiming to reflect an ‘average’ property of the forest. Except for stands that are thinned from above the dependence of stand height on stand structure and thinning practices decreases through the progression  $\bar{H}$ ,  $H_g$ ,  $H_L$ ,  $H_{100}$ , and  $H_{dom}$ . Consequently,  $\bar{H}$ ,  $H_g$ , and  $H_L$  are mostly useful for volume calculations, while  $H_{100}$  and  $H_{dom}$  are better suited for classification of site and stand productivity (Figure 8c).

## Volume

Volume is the most widely used measure of wood quantity. For some purposes, however, dry matter production is a more informative variable, but

because volume is more easily determined, dry matter content is usually derived from volume estimates. Alternatively, fresh weight may be used to quantify wood or dry matter, with or without adjustment for moisture content and wood density.

## Definitions

The wood volume of a tree includes stem, branches, stump, and roots (Figure 3). For standing trees, aboveground volume production is generally calculated on stemwood volume for conifers, but may include branch volume for broadleaved tree species. Depending on measurement objectives and local traditions measurements or predictions of wood cubic volume may refer to, for example, total stem volume,  $v$ , total tree volume (stem and branches),  $v_b$ , or volume above a certain merchantable limit ( $d \geq a$ ),  $v_a$ . Volume estimates may exclude or include bark (disregarding the fact that anatomically bark is not a wood component) and, for aboveground estimates, exclude or include stump.

## Measurement Principles and Instruments

Wood cubic volume may be determined by direct or indirect measurement. Direct methods are based on the principle of water displacement or pycnometry, while indirect methods are based on geometry. Indirect methods prevail because these are often more practical.

Generally, due to the tedious work, a much smaller sample of trees is measured for volume than for diameter and height. The volume of remaining trees is subsequently derived based on general or stand-specific models. Choice of sampling principles, sample size, and volume function should be based on requirements for end-use precision balanced with measurement costs.

It is relatively easy to sample and measure for stem volume, more tedious for branches, and quite difficult for stumps and roots. Stem volume can be measured on standing as well as felled trees, branch measurements are most often carried out on samples from the crown of cut trees, while stumps and roots that need excavation are only rarely measured. Consequently, and because of the commercial importance, methods and models for volume estimation mainly consider stemwood and other traditionally merchantable wood products.

Direct volume measurement is carried out using a xylometer, which is essentially a water tank in which the wood is submerged. The principle of water displacement requires that the xylometer is equipped or used with a suitably graduated volume scale. The cubic volume of wood equals the volume of water

and wood minus the volume of water. Pycnometry requires that the wood is weighed before and during submersion, for example on a scale beam or in a sieve bucket. The cubic volume of wood equals the apparent difference in mass.

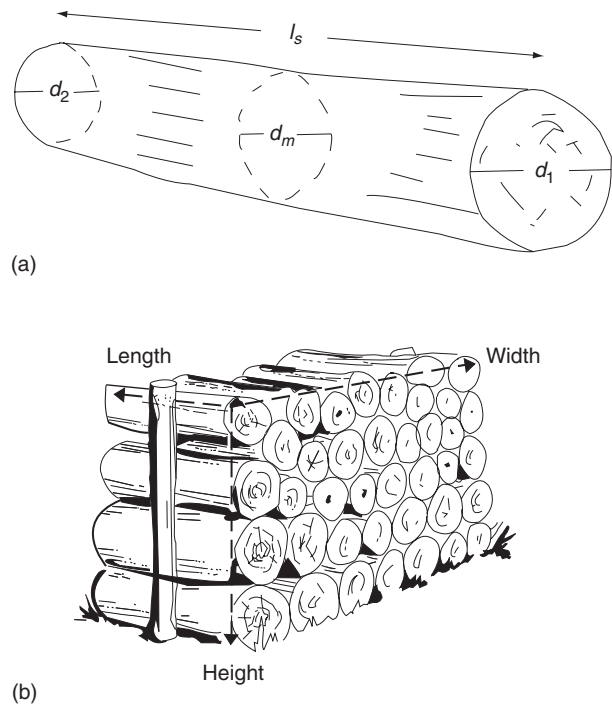
In principle, both approaches are accurate, but their precision depends strongly on a suitable graduation of measurement scales (i.e., relative to the size of the piece of wood being measured). Except for very small pieces of wood, measurement error due to air bubbles and the effect of water temperature is negligible. With pycnometry it is important that the water is still.

Direct volume measurement obviously requires destructive sampling. In scientific investigations it may be useful for measuring irregularly shaped parts of the tree and for testing the quality of indirect volume measurements. Often, branch wood is weighed as a substitute for xylometry. Commercially, direct volume measurements are used at some industrial-scale sawmills and plants, for example for scaling of pulpwood.

Indirect volume measurement relies on geometrical models of stem, branches, stump, and roots and may be carried out for felled as well as for standing trees. Due to differences in taper and shape different models are usually required for different parts or segments of the tree. The exact location and number of segments depend on measurement objective, species characteristics, tree size, and local measurement traditions. When felled trees are measured for total (aboveground) volume, for example for the construction of a volume function, the volume of under-cut and back-cut should be included or adjusted for.

In practice, the cubic volume,  $v_s$ , of each segment of a sample tree is calculated from the product of its cross-sectional area,  $g$ , and length,  $l_s$  (Figure 9a). Three formulae are commonly used for this purpose: Huber's formula with  $v_s = l_s \cdot g_m$ , Smalian's formula with  $v_s = l_s(g_1 + g_2)/2$ , and Newton's formula with  $v_s = l_s(g_1 + 4g_m + g_2)/6$ , where subscript  $m$  refers to the middle and subscripts 1 and 2 to one or the other end of the segment, respectively. The accuracy of each of these formulas depends on the shape of the segment (see section 'Form factor, stem taper, and volume models' below). Huber's formula provides relatively robust volume estimates for long logs and is often used in timber trading.

Most indirect volume measurements on felled trees are carried out using a caliper and a tape measure. Depending on circumstances, an optical caliper and a hypsometer may be better suited for standing trees. The measurement principles that apply to these instruments therefore also apply to the determination



**Figure 9** Measuring the cubic volume of wood and wood products. (a) Solid cubic volume is generally derived from measurements of diameter ( $d_1$ ,  $d_m$ , or  $d_2$ ) and length ( $l_s$ ) of logs and pieces of wood. (b) Stacked wood is often measured for stacked volume, whether in a regular stack (as here) or in an irregular pile (as for example a heap of wood chips). Stacked volume is the total space occupied by a stack or a pile of wood, as determined by its external dimensions. Solid wood content is calculated using a conversion factor that adjusts for open space in the stack.

of wood volume, and potential measurement errors compound to the calculation of cubic volume. Next, apart from sampling error, error in volume estimates may be due to the models used and, for volume at stand level, error that originates in determination of plot and stand area.

The measurement of trade volume for wood products depends on local customs and trade agreements. The trade volume of logs that are measured individually generally refers to solid wood content. Stacked wood is often sold by stacked volume and solid wood content derived using a conversion factor that adjusts for open space (Figure 9b). In both cases, trade customs often prescribe rounding down of measurements. Alternatively, wood products may be traded by weight, with conversion to volume or dry matter based on estimates of moisture content and wood density.

### Form Factor, Stem Taper, and Volume Models

The volume of stems, branches, stumps, and roots is modeled in numerous ways, using a range of different predictor variables. Here, three different

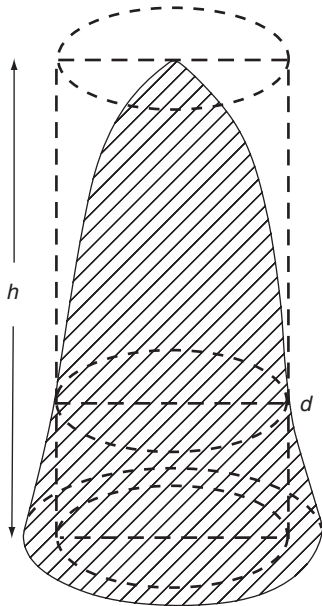


approaches to modeling stem volume serve as examples.

In its simplest formulation, the classical approach to estimation of volume,  $v$ , is based on three volume factors: basal area at breast height (or at another nominated reference level),  $g$ , total tree height,  $h$ , and a form factor,  $f$  (Figure 10). The product of basal area and height ( $g \cdot h$ ) yields the volume of the so-called reference cylinder. The form factor is a reduction factor, by which reference cylinder volume is multiplied to obtain the volume of wood. Thus, wood volume is calculated as  $v = g \cdot h \cdot f$ . Alternatively, diameter or circumference may enter the volume equation directly.

As a second example, volume may be estimated directly from dendrometric variables such as diameter and height. Classical volume models include the so-called combined-variable equation,  $v = \alpha + \beta \cdot d^2 h$ , and the more general model,  $v = \alpha \cdot d^\beta h^\gamma$ , where in both cases  $\alpha$ ,  $\beta$ , and  $\gamma$  are coefficients and other symbols are as usual. The latter model is often used after logarithmic transformation,  $\log v = \log \alpha + \beta \log d + \gamma \log h$ , where estimates of  $\beta$  often approach 2 while estimates of  $\gamma$  approach 1.

Choice of model may depend on modeling objective, data used in the estimation of coefficients



**Figure 10** The form factor,  $f$ . In one number, the form factor summarizes the volume of a tree relative to its height and its diameter or basal area at breast height, viz.  $f = v/(h \cdot d^2 \pi/4)$ . Counterintuitively, the form factor is not a characteristic of stem form. The form factor is generally influenced by tree species, age, site, and stand treatment. Because the reference point, for example 1.30 m above ground, is chosen arbitrarily rather than relative to tree size,  $f$  is called the artificial form factor. For conifers  $f$  usually refers to stemwood, whereas for broadleaved tree species  $f$  often includes branches.

and on error structure. Both models may be expanded to account for effects of, for example, age, site, and stand treatment. Apart from stand variables, the inclusion of an upper stem diameter often improves model performance. As an advantage, variation within stands can be quantified separately from variation between stands, and the accuracy of volume estimates improved. This requires, however, that there is no significant interaction between variables which determine volume level as this could result in ambiguous predictions due to lack of parallel response planes.

The classical form factor and volume equations can be used for individual trees as well as for stand values. Stand values are calculated by summation of individual tree or size class volumes, or a model tree is assumed. The model tree is sometimes referred to as the mean tree. Model tree dimensions reflect an 'average' tree of the stand in question. For example, with diameter corresponding to some stand average and height corresponding to some indicator of stand height. To obtain total stand volume, model tree volume is multiplied by stem number. Interestingly, the sum of individual tree volumes is identical to stand volume according to the mean tree method, when models of the combined-variable equation type are used with quadratic mean diameter,  $D_g$ , and stand height weighed by basal area,  $H_L$ .

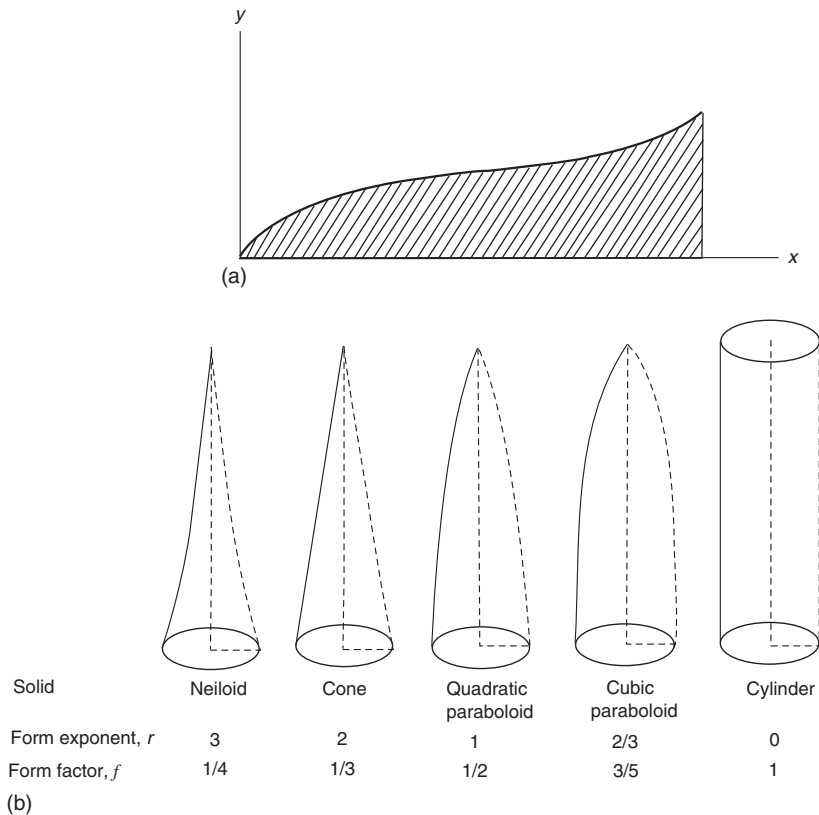
As a third example, stem taper and stem volume may be modeled jointly, resulting in compatible estimates of volume and diameter at any height along the trunk. In the compatible system of stem taper and volume functions, the solid of revolution of the stem taper equation equals the volume according to the stem volume function.

Assuming circular cross-sections (see section 'Diameter, girth, and basal area' above) the cubic volume of the stem or any segment of the tree can be calculated based on the general taper equation  $y = k\sqrt{x^r}$ , where  $y$  is the radius at any point along the length axis  $x$ , and  $k$  and  $r$  defines the rate of taper and the shape of the solid, respectively (Figure 11).

In forestry terms, the classic stem taper equation is often expressed as  $d_l^2 = 4p(h-l)^r$ , where  $d_l$  is the diameter at a given height  $l$  (above ground),  $h$  is total tree height,  $p = k^2 > 0$  and  $r \geq 0$ ;  $r$  is called the form exponent. The solid of revolution, i.e., the stem volume, is thus

$$v = \pi \int_0^h p(h-l)^r dl = \frac{\pi p}{r+1} h^{r+1} \\ = \frac{\pi}{4} d_{1.30}^2 \frac{1}{r+1} h \left( \frac{h}{h-1.30} \right)^r$$





**Figure 11** The classic stem taper equation and its solids of revolution. (a) The stem taper equation  $y = k\sqrt{x^r}$  provides a general model of radius,  $y$ , at any point along the length axis,  $x$ . (b) Resulting solids of revolution depending on the form exponent,  $r$ . The form factor,  $f$ , given for each solid refers to a cylinder of identical diameter at the base.

where  $v$  is cubic volume and other symbols are as just described. Combining these equations yields

$$d_l^2 = \frac{4v}{\pi b} (r+1) \left(1 - \frac{l}{b}\right)^r$$

When using the model in practice, some restrictions may be desirable. Apart from compatibility, predicted diameter at breast height should equal the measured diameter, predicted diameters should decrease with increasing height along the stem, no diameter predictions should assume negative values, predicted diameter at the tip of the stem should equal zero, the derivative of the stem taper function with respect to  $l$  should equal zero at the tip of the stem, and first derivatives should agree at points of transition along the stem.

With this model the stem volume is explicitly expressed in the stem taper function and it is possible to adjust estimates of the stem taper curve to a specific stand volume level, thereby accounting for the effects of age, silvicultural practice and site on stem taper. Compatible systems may be taper-based or volume-based, depending on which function is derived first.

When the value of  $r$  progresses from 0 to 3 the solid of revolution progressively changes from a cylinder ( $r=0$ ) through a cubic ( $2/3$ ) and a quadratic ( $1$ ) paraboloid to a cone ( $2$ ) and a neiloid ( $3$ ) (Figure 11b). Generally, stems approximate the shape of a truncated neiloid at the butt end, one or more truncated paraboloids in the middle, and a paraboloid or a cone towards the tip. Merchantable logs often falls between the frustum of a paraboloid and the frustum of a cone.

Considering the accuracy of indirect volume measurements (see section 'Measurement principles and instruments' above), all three commonly used formulas are accurate when  $r=0$  (a cylinder) and when  $r=1$ , and Newton's formula is unbiased for all of the geometric solids concerned. When  $r>1$  Huber's formula overestimates volume, while Smaalian's formula underestimates volume; vice versa when  $r<1$ . In practice, a stem and even a short wood segment may be an irregular composite of different geometric solids whose real shape usually remains unknown. Consequently, the theoretical bias should be interpreted with care.

The volume of branches is often modeled for whole trees at a time (rather than at a more detailed

level). Roots generally vary considerably and are very difficult to model.

As a rule of thumb individual stem volume can be estimated to within  $\pm 7.5\%$  of the true value and stand volume to within  $\pm 5\%$ . Often, the volume of individual trees is calculated to the nearest  $0.001 \text{ m}^3$  ( $0.0001 \text{ m}^3$  for very small trees) and at stand level to the nearest  $0.1 \text{ m}^3 \text{ ha}^{-1}$  for research plots and  $1\text{--}10 \text{ m}^3 \text{ ha}^{-1}$  in operational forestry.

### Applications

Quantification of wood volume and volume growth is one of the most important forest measurement activities. The wood volume of a tree or a forest stand is an integral measure of solid substance for a major component of the forest ecosystem and a potential basis for estimates of tree biomass, dry matter production, and carbon storage. Furthermore, trading of wood products is often based on their cubic volume, and standing volume and volume growth are significant decision variables in forest management. Importantly, the magnitude and quality of growing stock is closely correlated with the economic potential of the forest as well as with its biological and social value. At the national level, accurate inventory information on volume, volume growth, and their distribution to owner categories, growth regions, forest types, species groups, etc., is a basic requisite for suitable, well-targeted, and efficient forest policies.

As forest management objectives gradually change over time and may vary considerably from place to place, it is becoming increasingly important that forest measurement variables can be used purposefully for a wide range of different aims. It is easy to invent new and unproven variables, but hard to think of any variables that are more robust and less expensive to measure than those from which wood volume is derived. Although dendrometry rarely provides causal models for science or for forestry, it certainly should continue to provide suitable measurements for the sustainable management of our forests.

*See also:* **Biodiversity:** Biodiversity in Forests. **Experimental Methods and Analysis:** Biometric Research; Design, Performance and Evaluation of Experiments; Statistical Methods (Mathematics and Computers). **Health and Protection:** Diagnosis, Monitoring and Evaluation. **Inventory:** Forest Inventory and Monitoring; Large-scale Forest Inventory and Scenario Modeling; Modeling; Multipurpose Resource Inventories; Stand Inventories. **Landscape and Planning:** Spatial Information. **Mensuration:** Growth and Yield; Timber and Tree Measurements; Tree-Ring Analysis; Yield Tables, Forecasting, Modeling and Simulation. **Plantation Silviculture:**

Stand Density and Stocking in Plantations; Sustainability of Forest Plantations. **Resource Assessment:** Forest Resources; GIS and Remote Sensing; Non-timber Forest Resources and Products; Regional and Global Forest Resource Assessments. **Tree Physiology:** Shoot Growth and Canopy Development.

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## Timber and Tree Measurements

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### Introduction

Trees are complex three-dimensional structures that are very difficult to quantify or measure accurately. To overcome this complexity, the practice of tree measurement is generally to assume that portions of tree resemble simple shapes like cylinders, spheres, etc. A lot is known about these simple or Euclidean shapes, and this knowledge includes relationships between specific variables (e.g., radius of a sphere and the volume or surface area of that sphere). The closeness of the assumed shape to the real shape will partially control the closeness or the errors in