variables can be used to construct a range of simple yield tables for direct application to forest management. Summary parameters from these analyses, such as site index, can be used for classifying yield potential of real stands, either as part of stand management and production forecasting or as part of a more general inventory system. Sometimes, individual curves and equations developed to characterize particular growth variables are used to support inventory systems, for example in the form of volume tables or equations. Supplementary analyses may also be carried out that extend beyond direct assessment of stands, for example measurements of site index in stands may be analyzed with respect to site and environmental variables so that the potential growth of trees can be predicted for unplanted sites.

Models of increment are useful in a commercial context for very short-term forecasting. However, the main application of increment studies is in a research context, for example, for understanding relationships between tree and stand dynamics and environmental and management factors. As such, the study of increment has in effect developed into the science of dynamic growth modelling.

## Sample Plots

Mensuration research depends on high-quality, comprehensive data on the growth and yield of forest stands. Internationally there has been considerable effort over the past 150 years to collect such data either as isolated assessments or through the long-term monitoring of research plots (**Figure 6**). Such research plots are generally known as 'mensuration sample plots' and are categorized as either temporary (one-off measurement) or permanent (repeated, long-term measurements). The data obtained from sample plots have been vital to the understanding of forest growth dynamics, for mensuration research and for development of models of stand structure and yield (**Figure 7**).

## Future Developments in Mensuration Research

Although now a well established discipline, mensuration remains an important, arguably fundamental, element of forest research. Research will continue on development of inventory methods and growth and yield models but this is likely to be carried out in a more integrated context. For example, significant scope exists for integrating networks of mensuration sample plots with other forest monitoring networks addressing subjects such as forest condition, biodiversity, or carbon balance. Extension of the research to cover such integration would require the development of a more comprehensive range of measurement protocols and supporting equations and models. Rapid developments in the fields of geographic information systems (GIS) and remote sensing offer opportunities for combining traditional, intensive mensurational assessments with extensive, state of the art technologies such as satellite imagery.

*See also*: **Inventory**: Forest Inventory and Monitoring; Multipurpose Resource Inventories. **Mensuration**: Forest Measurements; Timber and Tree Measurements; Yield Tables, Forecasting, Modeling and Simulation. **Resource Assessment**: GIS and Remote Sensing.

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# Yield Tables, Forecasting, Modeling and Simulation

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## Introduction

Growth is usually defined as the net periodic annual increments of forest variables and the yield is their summation. Yield tables typically present the amount of forest variables from a given site, under certain prerequisities during a specific period. The period is usually the rotation, and the variables considered are basic forest parameters such as (mean) diameter, mean or dominant height, basal area, number of stems, and volume. Current and mean annual increment are also shown in a typical yield table, as well as yields obtained from thinnings, so that they should actually be called growth and yield tables. An example of a yield table is presented in **Table 1**.

Growth and yield predictions under different management options are a prerequisite in any forest management and planning task. The main purpose for developing the yield tables and yield equations has been to produce accurate predictions of forest yield to enhance decision-making. These models are typically empirical, based on observed development. Thus, although the object of all growth and yield studies is the past growth, their value lies in predicting the future development.

In this article, the historical development from first growth and yield tables to modern single-tree and projection models is briefly presented.

## Predicting Forest Growth and Yield

### Growth and Yield Tables

The very first growth and yield tables for pure stands were created in the late eighteenth and nineteenth century. In Germany, the number of publications about yield tables reached 1000 by 1880. An important type of table was the so-called normal yield table. In them, the yields of natural, fully stocked stands were shown. As most of the stands are not normal, they were utilized in yield predictions assuming that the relative density of the stand, compared to the normal stand, would remain the same. However, already in the 1930s this assumption was proved incorrect: the relative density was shown to approach that of the normal stands.

Empirical yield tables, on the other hand, presented the mean development of the stands with respect to age in cross-sectional (inventory) data for a large area. The measured stands were not required to be normal. In this way, the tables represent the development of average stands in the area. The tables were produced using graphical smoothing. They were not yet useful for planning the management of the stands, as they represented averages for different areas, sites, and treatment levels.

Later, site-specific yield tables were constructed from long-term experiments by grouping the observations according to site (**Figure 1**). However, site classification is a problematic field in itself. Climatic and soil properties are not easy to classify, and therefore, the observed yields are used as a measure of site quality. In early applications, quality classes based on mean height of the stands were commonly used.

**Table 1** Example of yield table of Scots pine

| T | H | N | G | V | $\bar{v}$ | $p_v$ | $I_v$ | $Y_v/T$ |
|---|---|---|---|---|---|---|---|---|
| 20 | 5.6 | 1800 | 6.3 | 19.0 | 11 | 24.6 | | 0.95 |
| 25 | 7.7 | 1800 | 11.1 | 42.4 | 24 | 14.7 | 4.7 | 1.70 |
| 30 | 9.7 | 1800 | 15.6 | 73.6 | 41 | 9.9 | 6.2 | 2.45 |
| 35 | 11.4 | 1800 | 20.0 | 110.2 | 61 | 7.4 | 7.3 | 3.15 |
| 40 | 13.1 | 1800 | 24.2 | 150.9 | 84 | 5.8 | 8.1 | 3.77 |
| 45 | 14.6 | 1800 | 28.2 | 194.5 | 108 | | 8.7 | 4.32 |
| 45 | 14.6 | 1033 | 19.4 | 136.1 | 132 | 5.2 | | 4.32 |
| 50 | 15.9 | 1033 | 22.9 | 171.4 | 166 | 4.2 | 7.1 | 4.60 |
| 55 | 17.1 | 1033 | 25.8 | 207.4 | 201 | 3.6 | 7.2 | 4.83 |
| 60 | 18.2 | 1033 | 28.8 | 244.2 | 237 | 3.1 | 7.4 | 5.05 |
| 65 | 19.2 | 1033 | 31.6 | 282.2 | 273 | | 7.5 | 5.24 |
| 65 | 19.2 | 628 | 21.8 | 197.6 | 315 | 3.1 | | 5.24 |
| 70 | 20.1 | 628 | 24.5 | 228.2 | 363 | 2.6 | 6.1 | 5.30 |
| 75 | 21.0 | 628 | 26.6 | 257.6 | 410 | 2.3 | 5.9 | 5.35 |
| 80 | 21.7 | 628 | 28.8 | 287.2 | 457 | 2.1 | 5.9 | 5.38 |
| 85 | 22.4 | 628 | 30.9 | 316.9 | 505 | 1.9 | 5.0 | 5.41 |
| 90 | 23.0 | 628 | 33.0 | 346.7 | 552 | 1.7 | 6.0 | 5.44 |
| 95 | 23.5 | 628 | 35.0 | 376.3 | 599 | 1.6 | 5.9 | 5.47 |

*T*, biological age (a); *H*, dominant height (m), *N*, number of stems; *G*, basal area ($m^2\,ha^{-1}$); *V*, volume ($m^3\,ha^{-1}$); $\bar{v}$, mean size of stems ($dm^3$); $p_v$, mean annual volume increment percentage of the future 5-year period (%); $Y_v/T$, mean annual volume increment up to the age in question ($m^3\,ha^{-1}$).

Scots pine: $H_{100} = 24\,m$, 100 years rotation, and two thinnings with 30% removal.

Reproduced from Vuokila Y and Väliaho HT (1980) Viljeltyjen havumetsiköiden kasvatusmallit. *Communicationes Instituti Forestalis Fenniae* 99(2): 271, with permission.

Nevertheless, other approaches, for example, based on mean annual increment were also applied. In recent years, the dominant or top height has been the most commonly used variable in quality classification (**Figure 2**). This is because it is assumed to be the stand characteristic least affected by thinning in the stands. In mixed and uneven-aged stands this might be a dubious assumption. For such cases, classification based on basal area index has also been proposed. In some countries, for instance in Finland, site classification based on ground vegetation has been used.

These provisional yield tables were density-free, meaning that an average intensity of thinning was assumed for all stands. Such tables could be used to compare managed and unmanaged stands, but for more detailed planning they were not very useful. In multiple yield tables, there might be two or three different management options, the yields of which could be compared.

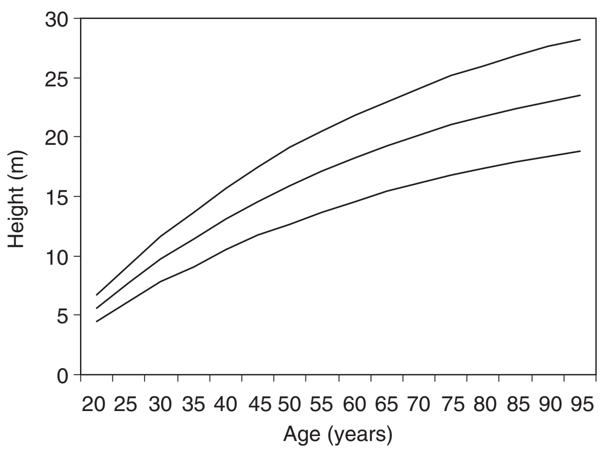


**Figure 1** Example of height–age models for site classification.

Even these multiple yield tables are problematic in forest management planning, as they are restricted to the few specific management options. Therefore, variable-density yield tables were needed. These tables are based on statistical models that can simulate the development of the stand under different conditions. Another approach, used particularly in north America, was explicitly to model the change in relative stand density, and to use this information together with normal yield tables to produce growth and yield predictions.

The development of yield tables was quite straightforward as long as pure stands were considered. Such yield tables have been used until recently in most countries, and in tropical countries they are still used. However, mixed stands have always been a problem for both yield tables and equations. The yield tables and equations can only represent a restricted number of different conditions. Therefore, in order to utilize the stand level models, a huge number of parameterizations would be required for different species mixtures, making the practical usefulness of the yield tables for mixed stands very limited. However, examples of such tables can be found.

### Growth and Yield Equations

For modeling the growth of stands, two types of models can be separated with respect to the time period considered. The first possibility is to predict the increment of basal area or volume in a period of a certain length, typically 5 years. The development is then calculated by summing subsequent predictions. This type is frequently used in Finland, for instance. Another possibility is to project the development of the desired variables, such as dominant height or basal area, for a desired time. These models, which

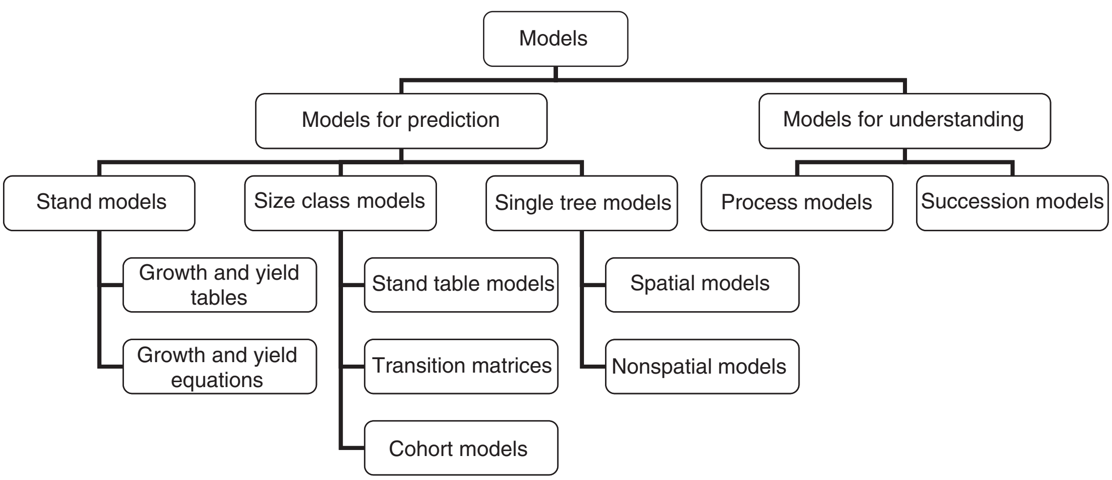


**Figure 2** The model classification. Simplified from Porté A and Bartelink HH (2002) Modelling mixed forest growth: a review of models for forest management. *Ecological Modelling* 150: 141–148, with permission from Elsevier.

are called projection models, are commonly used in north America and in tropical forestry.

At the beginning, equations were mainly used to construct yield tables. The first models typically predicted yield as a function of age. Assuming that the parameters of yield models depend on site index and stand density, their effect could be accounted for in the model. Uneven-aged or mixed stands were analyzed, for example, by classifying the data according to species composition and silvicultural history, and by estimating a different parameter set for each group.

Nevertheless, yield equations always assume a certain management regime throughout the projection, but growth equations allow treatments to be defined as desired. The models published by Vuokila for Scots pine (*Pinus sylvestris*), for example, predicted the growth percentage of basal area, dominant height, and volume as a function of stand age, dominant height, basal area, percentage of basal area removed in thinning, and mean diameter. The growth percentage was defined for growth between two successive thinnings.

In those early days, growth and yield models were constructed independently of each other. Therefore, summing up the growths might not produce the predicted yield. To obtain compatible growth and yield estimates many forms of differential equations have been used. Such growth function can be integrated to obtain the yield and growth function can be obtained as a derivative of the yield function. An example of such models is the famous Chapman–Richards equation where growth is described by the equation:

$$\frac{dx}{dy} = k\left(\frac{A-y}{A}\right)y$$

and the yield by the equation:

$$y = B(1-e^{-kx})^{1/1-m}$$

where $k$, $A$, $B$, and $m$ are parameters. In this model type, only one independent variable can be included, typically the stand age.

Obtaining compatible growth and yield equations for several variables from multiple regression models was a challenge that has been tackled from the early 1960s. For example, the equation system proposed by Clutter is:

$$\begin{aligned} E[\ln(V_1)] &= \beta_0 + \beta_1 S + \beta_2 A_1^{-1} + \beta_3 \ln(B_1) \\ E[\ln(V_2)] &= \beta_0 + \beta_1 S + \beta_2 A_2^{-1} + \beta_3 \ln(B_2) \\ E[\ln(B_2)] &= (A_1/A_2)\ln(B_1) + \alpha_1(1 - A_1/A_2) \\ &\quad + \alpha_2(1 - A_1/A_2)S \end{aligned}$$

where $V$, $B$, $A$, and $S$ denote volume, basal area, age and site index, respectively, and $\beta$ and $\alpha$ are parameters. The subscript in stand variables denotes the time.

In order to obtain compatibility, the system of equations was solved so that $\ln(B_2)$ was substituted with $E[\ln(B_2)]$ in the function of $E[\ln(V_2)]$. Therefore, parameters for projecting the basal area could be solved from parameters of the volume functions. The volume equation can be differentiated to produce a growth function:

$$\frac{dV}{dt} = \beta_2(V/B)\left(\frac{dB}{dt}\right) - \beta_3 V/A^2$$

This formulation led to compatible equations for standing volume, basal area growth, volume growth, predicted basal area, and predicted stand volume.

This approach is heavily based on stand age as an independent variable. Therefore, it is not suitable for uneven-aged stands, where the definition of stand age is at best ambiguous. For uneven-aged stands, a formulation based on stand density (defined by basal area) was introduced. A following system of equations, also providing compatible estimates, was used:

$$\begin{aligned} V &= \beta_0 B^{\beta_1} \\ \left(\frac{dV}{dt}\right) &= \beta_1 V B^{-1}\left(\frac{dB}{dt}\right) \\ \left(\frac{dB}{dt}\right) &= nB^m - kB \end{aligned}$$

In this approach, the parameter estimates are inefficient in statistical respect, i.e., estimates with better statistical quality could be derived. Therefore, in later applications, simultaneous estimation methods such as two- and three-stage least-squares and seemingly unrelated regression have been used. With these methods, compatibility can be achieved between the prediction and projection models in the system of equations.

### Utilizing Size Class Models

**Current yield** Later, the growth and yield equations were used to construct stand growth simulators for computers. After that it was possible to compute yield tables for any desired management schedules that were computable. These first stand simulators, however, could not produce enough information for modern forest management. There was a growing demand for single-tree information. For example, the timber assortments and value of the stand are difficult to predict with stand information.

Therefore, information concerning the frequencies in different diameter classes, i.e., diameter distribution, was strived for. The current yield is often predicted using probability density functions such as normal, log-normal, beta, gamma, Johnson's $S_B$ or Weibull. For Weibull, an analytical cumulative distribution is available. It describes the probability of values smaller than $x$. It is of the form:

$$F(x) = 1 - \exp\left[-\left(\frac{x-a}{b}\right)^c\right]$$

The proportion of trees in any diameter class $[d_1, d_2]$ can be calculated from the distribution function $F$ as $F(d_2) - F(d_1)$. The diameter distribution can also be formulated with respect to basal area. Then, the distribution gives the proportion of stand basal area in desired diameter classes. It is a weighted version of diameter distribution, which gives more emphasis to the most valuable trees in the stand.

For predicting the distribution, two main methods have been applied, namely the parameter prediction method (PPM) and the parameter recovery method (PRM). In PPM, the parameters of some distribution function, for example the Weibull distribution, are predicted with regression models from measured stand characteristics. In this approach, site index and age may be quite poor predictors of the parameters, but mean (or median) diameter usually gives a fairly good fit. In PRM, the parameters of the distribution function are solved from a system of equations, equating (measured or predicted) stand attributes to their analytical counterparts. The characteristics can be, for example, percentiles or moments of diameter distribution. In some cases, some of the parameters are predicted and others are solved using a parameter recovery approach.

Another possibility is to use a percentile-based diameter distribution method. In this method, a number of percentiles (of frequency or basal area) across the range of diameters are defined. Usually 12 or more percentiles are used. The distribution is obtained either by linear interpolation (i.e., assuming a uniform distribution of frequencies between adjacent percentiles) or any other monotony-preserving interpolation method. Recently, the distributions have also been predicted using nonparametric approaches. Then, the tree list is obtained as a weighted mean of tree lists from measured stands similar to a target stand.

The predicted diameter distribution is usually scaled to the measured number of stems, so the stem number obtained from the distribution corresponds to the known characteristic. Using the parameter prediction method or nonparametric approach, there is no guarantee that other stand characteristics obtained from the predicted diameter distribution correspond to the measured stand characteristics. With PRM, the compatibility of predicted and measured stand characteristics can be guaranteed for the characteristics used for solving the parameters of the diameter distribution function. Recently, calibration or adjusting techniques have been proposed for such situations.

**Future yield** When growth of a stand is predicted via size class models, both the current and future stand table are predicted, and the growth is calculated implicitly from the differences between the yields obtained from these tables. A simple way to accomplish such predictions is to predict the parameters of a probability distribution as a function of age and site for desired time points. Another possibility is (simultaneously) to project variables such as number of stems, basal area, and dominant height, and to predict the distribution in the future based on these variables. Then, however, the diameter distributions at different time points are not necessarily compatible.

To obtain a logical development of the distribution, the changes in the diameter distribution are directly predicted. This can be done, for example, by projecting the development of the parameters of the used probability distribution (PPM) or the development of percentiles of diameter distribution for a given time. In the latter case, the future distribution is obtained by analytically solving the parameters of the probability distribution from these percentiles (PRM) or by interpolating between the predicted percentiles (percentile-based method).

It is also possible to project the stand table directly. Then, the whole tree list is assumed to be known. The development of the list is based on an assumption that the relationship between the basal area of a tree and the average basal area follows a certain function. The stand table is constrained so that the projected number of stems and the basal area are consistent with the whole stand estimates. A mortality function is an important part of this system. Such models are already approaching modern single-tree models.

In the matrix approach, the stand is also described with the aid of size classes. These models, however, are stochastic. The model predicts the development of the stand via the probability that a tree will grow up to the next size class, die, or remain in its current class. Therefore, matrix models implicitly include models for recruitment and mortality. The probabilities are usually assumed to depend only on the current size of the trees. The results of these models are the frequencies of trees in different size classes.

The matrix models used are referred to as Usher, Lefkovitsch, or Leslie matrices depending on their characteristics: Leslie used age classes for animal populations, Lefkovitsch used development classes for insects, and Usher used diameter classes for forests. If the probabilities of movements are expressed as a matrix **M**, and the initial and final state of the stand with vectors $\mathbf{V}_0$ and $\mathbf{V}_1$, the prediction for one period is obtained as

$$\mathbf{V}_1 = \mathbf{M}\mathbf{V}_0$$

and for $n$ periods as

$$\mathbf{V}_1 = \mathbf{M}^n\mathbf{V}_0$$

It has been argued that the maximum exploitation of the stand and the stable stand structure can be revealed from the first eigenvalue and eigenvector of matrix **M**. However, they cannot be used to evaluate the optimal density of the stand.

One obstacle in using matrix models for growth prediction is that the basic models do not allow for probabilities to change in time, e.g., as a function of stand density or structure. This problem can be avoided, however, by estimating a new matrix for each iteration using equations.

## Assessing the Accuracy of Predicted Forest Development

Models need to be evaluated before they are used in real-life applications. Evaluating growth and yield models requires both qualitative and quantitative analysis. The qualitative analysis of the logics and biological consistency are as important as quantitative analysis of statistical properties. However, the models cannot be proved correct in evaluation: they can only be validated with respect to their usefulness in the applications for which they are meant.

The accuracy of different growth and yield models has been assessed with empirical validation studies. The simpler the models are, the more often they are also validated. The stand level models have generally performed better in these tests than the tree level models, due to the cumulating errors in the tree level. This concerns the short-term predictions — in the long term there are very few validation studies. Consequently, the long-term results may be much worse than expected.

The empirical accuracy assessments based on validation studies are, however, calculated for certain past time periods and for a certain area. To anticipate the precision and accuracy of future predictions, a model-based approach is required. It is possible to model directly the observed past errors of interesting variables predicted using the simulator.

There are also methods for assessing the accuracy of predictions which are not based on empirical validation studies. For instance, the precision of long-term predictions has been assessed through Monte Carlo simulation or Taylor series approximations, where the total prediction error is composed of several error sources. These methods can be applied in producing error budgets for simulators. Such budgets give the contribution of each and every error source in the results. On the other hand, it is difficult to take all sources of error into account. For example, the errors in the model structure, causing biased predictions, are difficult to incorporate into these methods. Yet, in the long term, the model misspecifications may be the most important source of error.

## Data for Growth and Yield Modeling

For standwise growth and yield tables, data not identifying individual trees are sufficient. The simplest estimate for growth is the difference between the volumes from two points of time. However, accounting for mortality and recruitment requires more information. Net growth can be defined as:

$$\Delta V = V_2 - V_1 + V_c$$

where $V_c$ is the harvested volume. Gross growth is obtained by adding mortality $V_m$ to the net growth. The growth of the trees observed at both time points is defined as survivor growth.

Permanent, temporary, and semitemporary plots have been used for estimating yield tables and equations. The first tables were based on temporary plots, but permanent plots have later been used in most countries. The obvious explanation for this is that temporary plots do not provide a good basis for analyzing the effect of different management options. However, at the time the first permanent experiments were established, statistical principles such as replications were not known. This somewhat reduces the value of the experiments.

If a retrospective analysis of the development of the stands can be accomplished, temporary plots may, however, be a fairly good option. Temporary plots are cheap and fast to measure, in contrast to permanent plots. In temporary plots the measurement personnel, equipment, and calculation techniques do not vary in the data. Furthermore, the studied treatments are not restricted to a few possibilities, and they can be chosen to be up-to-date options. In temporary plots, natural damages do not affect the studies. Nevertheless, the treatments carried out before the measurements are difficult to define afterwards.

Semitemporary data remeasured at fixed intervals may be a suitable compromise for most occasions. However, the remeasurements should cover a sufficient period in order to include the whole range of variation due to weather conditions. It is also important to cover the extreme densities and treatments in the data. The remeasurement intervals also need to be long enough to ensure that growth can be detected from noise introduced by measurement errors.

## Final Remarks

Forest simulators have been developed from simple standwise yield tables and models to increasingly complex single-tree models. The new models are more flexible and suitable for many applications, for mixed stands and even for changing management practices. The causal relationships governing the growth of forests are easier to account for in single-tree models than is a stand model. However, the accuracy of the predictions has not been improved likewise. On the contrary, the more complex a simulator is, the more uncertain the predictions may be.

All in all, it is important to study the contribution of different sources of uncertainty (i.e., to formulate error budgets) to concentrate research efforts where they are most needed. It may well be that the main source of uncertainty is not the growth simulator at all, but the quality of initial data, for example. It would also be useful to separate the inherent randomness of growth from pure ignorance.

*See also*: **Experimental Methods and Analysis**: Statistical Methods (Mathematics and Computers). **Inventory**: Modeling. **Mensuration**: Forest Measurements; Growth and Yield

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# Tree-Ring Analysis

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## Introduction

Tree-ring analysis or dendrochronology is both an old and a modern science. Just counting tree rings sounds simple, but in the context of forest dynamics tree age is an important and valuable parameter. The pattern of tree-ring width, wood density, element content, and other features store information on past growth conditions. Biomonitoring is the reflection of growth factors by biological organisms and their