# KINEMATIC FLOW THEORY

### INTRODUCTION

Hydrological methods used regularly in practice for the design of stormwater drainage systems include the rational method and other isochronal methods. That is, they assume each point in the catchment has a unique travel time to the mouth. These methods do not account for storage in the system and the time variation in flow rates. Storage may occur on the surfaces of vegetation, roofs, walls, on the ground, in depressions and in channels. The storage may be permanent (retention) or temporary (detention). The relationship between flow depth and discharge and the effect of surface friction of the catchment are not accounted for either. The rational method and the isochronal methods are based on the assumption of uniform flow throughout the catchment. Concentration times in channels are calculated from steady-state water velocities and the dynamics of the system are not accounted for. In fact, there is a gradual increase in depth of flow with time at any point on the catchment and the depth of flow gradually increases down the catchment. The flow is therefore both unsteady and non-uniform. The time to equilibrium is therefore a function of rate of precipitation and it is not necessarily the travel time down the catchment. The kinematic method accounts for these factors in a simplified manner. It also accounts for catchment slope, roughness and infiltration.

Flow rate and velocity are related to depth according to the discharge relationships. In fact even the assumption of the steady state depth-discharge function can in some cases lead to error. Momentum and energy balance are necessary for a true representation of flow conditions through the system. Nevertheless, the full differential equations of motion, termed the Navier-Stokes equations, are complex and not warranted in most circumstances. Even the one-dimensional St. Venant equations can be simplified in many instances as will be illustrated later.

### EQUATIONS OF MOTION

The differential equations describing one-dimensional flow in open channels may be derived from consideration of continuity and momentum balance. The following assumptions are made at this stage:

- (i) Flow is one-dimensional i.e. in one direction. Acceleration in the directions perpendicular to the flow direction is therefore neglected.
- (ii) The pressure at any depth is the hydrostatic pressure.
- (iii) Depth is constant across any section, i.e. the channel is rectangular.
- (iv) Momentum transferred to the flow from lateral inflow is negligible.
- (v) The fluid is incompressible.
- (vi) The uniform flow friction equation applies to non-uniform and gradually varied flow.
- (vii) The bed gradient is small, so that  $\theta = \tan \theta = \sin \theta = S_0$
- (viii) Velocity is constant across any section.

The continuity equation may be derived by considering the balance of flow across the boundaries of an element such as in Fig. 4.1.



Fig. 4.1 Continuity of flow.

Equating inflow to outflow plus increase in storage produces directly  $\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q_i$ (4.1) where Q is the flow rate, A is the cross sectional area,  $q_i$  is lateral inflow per unit length in the x direction and t is time.

A dynamic balance is obtained by equating the unbalanced force across an element to the acceleration and change in momentum across the element as illustrated in Fig. 4.2. Thus, since  $F_n = w\bar{y}A$  and  $F_s = wA S_f dx$ , we have

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 $\frac{\partial}{\partial x} (w\bar{y}A) dx + wA S_{f} dx - wA S_{o} dx = -(w/g) \frac{\partial}{\partial x} (Av^{2}) dx - \frac{w}{g}A \frac{\partial v}{\partial t} dx - \frac{w}{g} q_{i}v dx$  (4.2)Re-arranging this,  $g \frac{\partial (\bar{y}A)}{\partial x} + \frac{\partial (Av^{2})}{\partial x} + A \frac{\partial v}{\partial t} = Ag (S_{o} - S_{f}) - q_{i}v$  (4.3)If the channel is rectangular and flow is nearly uniform, this simplifies to

$$\frac{g_{\partial Y}}{\partial x} + v_{\partial x}^{\partial V} + \frac{\partial v}{\partial t} = g (S_o - S_f) - q_i v/A$$
(4.4)

Equations (4.1) and (4.4) were first published by St. Venant (1848). These can be solved numerically for various cases of unsteady flow in open channels. The equations are not simple to solve. The inclusion of the terms for acceleration and change in momentum are in some cases not worthwhile, in which case solution of the equations is simplified.



Fig. 4.2 Momentum balance in flow direction.

In the above equations, F is a force, y is the depth of flow,  $\overline{y}$  is the depth of the centroid, S<sub>0</sub> is bed slope, S<sub>f</sub> is friction gradient, g is gravitational acceleration, w is the unit weight of fluid, and v is the flow velocity.

## KINEMATIC EQUATIONS

In many cases of overland flow and even open channel flow, the momentum change and acceleration terms can be neglected. In fact this is equivalent to assuming that the energy line is parallel to the bed, i.e.  $S_f = S_o = S$  (4.5) It is not difficult to imagine this holds in the case of overland flow. Over a distance of 100 metres the ground surface may fall of the order of one metre whereas the difference in depth of flow over the same distance will only be of the order of a millimetre. The corresponding difference in velocity head may be of the order of 10 mm. The resulting simplified equations are termed the kinematic equations. They are equivalent to the assumption of unsteady uniform flow.

Most equations for friction gradient can be written in a form relating depth of flow y to flow per unit width q, with an equation of the form  $q = z y^n$ . (4.6)

Here z is a constant involving g,  $S_f$ , and the fluid and surface properties. For laminar flow n is unity, while for turbulent flow in rough conduits n is 2. Overton and Meadows (1976) indicate that in the case of overland flow the flow can be turbulent a very low Reynolds numbers (transition  $R_e = iL/v = 20$  to 2000). This is largely due to the effect of rain falling on the surface. Here i is the rainfall rate, L is the length of flow path and v is the kinematic viscosity of the fluid, water For rough surfaces the popular Manning equation indicates a value for n of 5/3 and z is  $KS^{\frac{1}{2}}/N$ , where N is Manning's roughness coefficient and K is 1 in SI units and 1.486 in fps units. If the Manning roughness N is approximated by  $0.13Kk^{V_6}/g^{V_2}$  the dimensionless Manning-Strickler flow equation results:

$$q = 7.7 (Sg)^{\frac{1}{2}} y^{\frac{5}{3}} / k^{\frac{1}{2}}$$

(4.7)

(4.8)

The discharge equation must be considered together with the continuity equation (4.1) which when expressed in terms of flow per unit width, q, becomes

 $\frac{\partial q}{\partial x} + \frac{\partial y}{\partial t} = i_e$ 

where the excess rainfall rate  $i_e = i - f$  and i is the rainfall rate and f is the loss rate, all per unit area.

## SOLUTION OF THE EQUATIONS FOR OVERLAND FLOW

General solution of the two equations (4.6) and (4.8) is possible for various situations of overland flow and sometimes conduit flow. Numerical solutions by computer are comparatively easy (Wooding, 1966; Constantinides and Stephenson, 1981) and many computer models are based on these simplified equations instead of the more rigorous differential equations. In many situations the equations are satisfactory even for conduit flow. This is the case for relatively steep gradients, but where backwatering and rapid changes in flow or gradient occur, the rigorous equations must be employed.

Woolhiser and Liggett (1967) analyzed the rising limb of an overland flow hydrograph using both the kinematic and the complete equations. Their results indicate that the kinematic form of the equations is reasonably accurate for  $gh/v^2$  greater than about 10 (h is the elevation difference down the length of the catchment and v is the equilibrium flow velocity at the end of the catchment).

The value of the kinematic method lies in the feasibility of obtaining analytical solutions. Thus expressions describing the shape of a hydrograph and the concentration time for different cases of overland flow and channel flow were derived by Wooding (1965) and others.

The classical method of solution of the equations for overland flow is by the method of characteristics (Henderson, 1966; Eagleson, 1970). This method involves the substitution of total differentials for partial differentials while integrating along a so-called characteristic line where x is related to t. Elements of the method can be explained without a complete mathematical background. This is attempted in the following section in order to introduce the reader to some of the resulting solutions which have been achieved.

Equations (4.6) and (4.8) may be used to derive some useful relationships in the case of constant excesses rainfall rate i\_. Recall that  $q = zy^n$ (4.6) $y = (q/z)^{1/n}$ Then (4.9)Now put  $\frac{dx}{dt} = \frac{dq}{dy}$ =  $nzy^{n-1}$ = nq, z(4.10)(4.11)(4.12)(i.e. consider an imaginary wave travelling at a speed  $\frac{dx}{dt} = n \ z \ y^{n-1}$  down the basin). Then the total differential  $\frac{dy}{dt} = \frac{\partial y}{\partial t} + \frac{\partial y}{\partial x} \frac{dx}{dt}$  $= (i_e - \frac{\partial q}{\partial x}) + \frac{q^{1/n-1}}{n_z^{1/n}} \frac{\partial q}{\partial x} nq^{1-1/n_z^{1/n}}$ (4.13)(4.14)(4.15)= i\_ i.e. the depth of flow increases at the rate of excess rain when travelling at the speed  $\frac{dx}{dt} = nzy^{n-1}$  down the basin. Since Since  $\frac{dy}{dt} = i_e$ it follows that  $y = i_e t$ (4.15)(4.16)

Hence	$\frac{dx}{dt}$	= $nz(i_et)^{T}$	n <b>-</b> 1		(4.1	7)
Integrating:	x	$= zi_0^{n-1} t^1$	n		(4.1	8)
Thus water s	tarting a	t thể begin	nning of the	storm at	the top of t	he
basin and tr	avelling	down the ba	asin until x	= $L$ (the	length of th	e
basin) will	reach x =	Lata tir	ne given by			
$t_c = (L/zi_e^n)$	-1 <sub>)</sub> 1/n				(4.1	9)
and then q	= zy <sup>n</sup>				(4.2	0)
	= z(i <sub>e</sub> t	.) <sup>n</sup>			(4.2	1)
	$= zie^{n}L$	/zien-1			(4.2	2)
	= Lie	-			(4.2	3)

Thus the outflow equals the input at this point in time and there must be equilibrium after this time. Thus  $t_c$  is the concentration time of the basin. Note that it is not a function of the final equilibrium velocity at the mouth, but depends on the rate of flow buildup along the basin. In fact, the speed of propagation at any point is nv where the water velocity v = q/y (4.24)  $= z y^{n-1}$  (4.25)

The relationship between wave celerity and flow velocity may be seen by comparing (4.11) and 4.25).

Thus the travel time at equilibrium is greater than the concentration time by the factor n (1.67 in the Manning equation). The practice of assuming that travel time equals storm duration for maximum peak runoff can therefore be unsafe as it results in an underestimate of design storm intensity.

There is some confusion between concentration time, lag time, time to equilibrium and travel time, and various authors have adopted different interpretations. The following definitions are used throughout this text:

Time to equilibrium,  $t_e$  is the time from the commencement of the storm to equilibrium flow conditions at the discharge point. It may be shown that for a plane  $t_e = (Li_e^{1-n}/z)^{1/n}$  (4.19) Travel time,  $t_t$ , is the time it takes for a drop of water to proceed from the most remote part of the catchment to the discharge point. The water velocity varies in time and space, and the custom is to assume steady-state flows and varying velocities longitudinally. It equals inlet time (from overland and roofs) plus flow time in drains. It was shown that for a plane  $t_e = (1/n)t_t$ .

*Concentration time*, is here defined as the time it takes for the flow to become steady at the discharge point. It is therefore equal to the

Overton and Meadows (1976) on the other hand detime to equilibrium. fine it as the travel time. Lag time,  $t_{I}$ , is the time between 50% of the rainfall and 50% of the runoff. It may be shown that for a plane  $t_{L} = \{n/(n+1)\}t_{e}$ . Storm duration,  $t_d$ , is the time from the commencement of the storm to the end of precipitation. An expression for the discharge at the mouth of the basin is obtained as follows: At any time before the limiting characteristic (from the top end of the basin) reaches the mouth, then at the mouth,  $\frac{\partial q}{\partial x} = 0$ (4.26)This is because the depth at every point other than influenced by the upstream boundary increases uniformly at a rate i\_. Thus  $y = i_e t$ (4.27)and from (4.6)  $q = z(i_0 t)^n$ (4.28)If the rain stops at some time  $t_d$  at or after  $t_c$ , then an expression for the falling hydrograph leg may be derived as follows: Since for equilibrium at any point before the rain stops  $q = xi_{\rho}$ (4.29)where  $\tilde{x}$  is the distance from the top end of the basin then from (4.6),  $x = zy^n / i_o$ (4.30)After the rain stops dy/dx = 0 and surface water will flow down the basin at a constant speed  $dx/dt = nzy^{n-1}$ Thus x = x<sub>0</sub> +  $\frac{dx}{dt}$  (t-t<sub>d</sub>) (4.11)(4.31)=  $zy^{n}/i_{e} + nzy^{n-1}(t-t_{d})$ (4.32)At the end of the basin x = L so the discharge is given by the implicit relation  $L = q/i_e + nq^{1-1/n}z^{1/n}(t-t_A)$ (4.33)The shapes of the rising and falling legs of the discharge hydrograph

are illustrated in Fig. 4.5 for various cases. Three cases are illustrated. For comparison purposes the total amount of rain has been assumed constant but the storm duration is varied. Thus for a storm which stops before  $t_c = (L/zi_e^{n-1})^{1/n}$ , spatial equilibrium will not be reached. The rising limb will follow Equ. 4.28 until the rain stops. Then discharge will remain constant since depth will be the same over a length of basin until the effect of the upstream basin boundary reaches the mouth. Then the hydrograph will fall. The case when  $t_d=0$ is that of the instantaneous hydrograph. For the limiting case when rain duration equals the concentration time, the falling limb will immediately follow the rising limb. For longer duration storms the system will reach an equilibrium after  $t_c$  and the outflow will only fall when rain stops. If the rain stops before  $t = t_c$ , then the spatial equilibrium condition would not occur and the hydrograph would tail off at an earlier stage as illustrated in Fig. 4.5.



Fig. 4.3 The runoff plane with precipitation

EFFECT OF INFILTRATION

Although the preceeding analysis allowed for losses during the storm, it was assumed that uniform losses ceased when rainfall ceases. In fact infiltration will continue as long as there is water on the surface.

Wooding (1966), produced an analytical solution for the shape of the hydrograph for different cases for n = 2. For storm durations greater than the concentration time, the rising limb and equilibrium discharge are as described previously. After the storm stops, we have  $y = y_0 - f(t-t_d)$  (4.34) In a similar way to (4.33) it may be shown that  $yL = (i-f)^{\frac{1}{2}} \{ i(t-t_d)^2 + t_c^2(i-f) \}^{\frac{1}{2}} - i(t-t_d)$  (4.35) The outflow stops at  $t = t_d + t_c(i-f)/(if)^{\frac{1}{2}}$  (4.36)

For storm duration less than concentration time, the falling limb will have two components. Initially the flow depth will decrease uniformly until the upstream boundary effect reaches the mouth, then the tail will decrease exponentially. The resulting different hydrograph shapes are depicted in Fig. 4.6.



Fig. 4.4 Water depth along catchment



Fig. 4.5 Outflow hydrograph shape for different storm durations but similar total excess rain



Fig. 4.6 Effects of infiltration on catchment discharge

CATCHMENT - STREAM MODEL

The assumption of a plane with uniform flow across the width is seldom a true picture. A catchment usually slopes down towards a centre channel. A more accurate representation appears in Fig. 4.7. Even this is a simplification, as overland flow will actually have a component towards the mouth and not just perpendicular to the stream.

In cases where overland flow time is negligible (e.g. a long narrow catchment with a small longitudinal fall), the channel may be taken as the catchment, with excess rain per unit length of channel equal to that per unit length of catchment. In many cases the overland flow time is not negligible and the runoff relationship becomes more complicated than for overland flow.

Analytical solutions are not feasible and numerical analysis was employed by Wooding (1965), to produce stream hydrographs.

The ratio of concentration time of the stream or channel  $t_s$  to the concentration time of the overland flow  $t_o$  per uniform stream inflow is defined as T.

Fig. 4.8 depicts the stream discharge hydrograph for different concentration time ratios. The discharge rate is expressed as a function of the uniform excess rainfall rate  $i_e$  and the total catchment area  $A_c$ . Time is expressed in terms of the overland catchment concentration time  $t_o$ . The parameter is the ratio of storm duration  $t_d$  to overland catchment concentration time  $t_o$ .



Fig. 4.7 The catchment-stream model



Fig. 4.8 Hydrographs for the catchment-stream model

## SOLUTIONS FOR TYPICAL RAINFALL INTENSITY-DURATION CHARACTERISTICS

Rainfall data for various stations throughout the world have been analyzed to yield depth of precipitation versus storm duration and return period. The form of the results varies with the analysis and the mathematical distribution selected but essentially the average intensity of precipitation decreases with storm duration for any selected recurrence interval or return period. It is this fact which results in non-linearity between catchment area and peak runoff rate for any recurrence interval. The time of concentration, or time to peak, for any catchment is a function of the catchment size amongst other things. So it is apparent that the larger the catchment, other factors remaining constant, the longer will be the storm duration resulting in maximum runoff, even though intensity of rainfall is bound to decline the longer the storm duration, other factors being equal.

Here kinematic methods are employed together with a generalized rainfall distribution equation to estimate concentration times and peak runoff rates for a range of catchments. By rendering the results dimensionless, they may be applied globally. It is necessary to assume a rectangular catchment and uniform storm distribution. Numerical techniques must be employed for non-uniform and time-varying storms.

An attraction of the kinematic approach is that all the variables are physically measureable. No empirical factors are required. The slope, roughness and length of catchment are all measureable, although an approximate equation for friction gradient is employed. Infiltration and initial losses are still difficult to assess, but the U.S. Soil Conservation Service (SCS, 1972) has given guidelines.

The present study can be applied to cases of uniform infiltration or an initial surface retention. A combination of these will approximate to a diminishing loss with time i.e. a decay in loss rate.

 $t_c$  is referred to as the concentration time of the catchment. It was observed previously that it is a function of the catchment characteristics as well as the rate of excess rain  $i_e$ . It is therefore necessary to solve for time of concentration as a function of excess rainfall rate, which in turn is a function of storm duration for any locality and return period. The rain is assumed uniform in time and space, but the losses may not be so. Initial retention may absorb some of the rain and infiltration may vary with time and antecedent conditions.

In the following analysis two simplistic loss models are employed in deriving analytically the concentration times and runoff for rectangular catchments. One model assumes all the losses to occur at the beginning of the storm, as would occur for catchment storage. The other model assumes a uniform rate of loss for the entire storm duration. Combinations of the two types of loss may be interpolated between the two extremes, which are plotted on accompanying charts.



Fig. 4.9 Typical rainfall intensity-duration relationship

STORM INTENSITY - DURATION RELATIONSHIPS AND SOLUTION FOR TIME OF CONCENTRATION

For any particular locality and recurrence interval, there is a statistical relationship between storm duration and intensity. Analysis of storms through the world indicate that the storm intensity may be

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predicted with reasonable accuracy with an equation of the form

 $i = \frac{a}{(b + t_d)}c$ (4.37) where a, b and c are regionally applicable constants. Overton and Meadows (1976) and Stephenson (1980) found c to be approximately unity in some cases. Thus in many situations it is possible to approximate the relationship by one of the form

$$i = \frac{a}{b + t_d}$$
 (4.38)  
where  $t_d$  is the storm duration, and 'a' is a function of the locality  
and return period. It was proposed by Lloyd-Davies that the maximum  
peak runoff for any recurrence interval will occur if the storm lasts  
only as long as it takes to reach equilibrium conditions. A longer  
duration storm will be of lesser intensity and a shorter duration storm  
will not reach equilibrium. It is now realized this is not always the  
case. The maximum peak runoff for abnormal basin shapes can occur from  
a storm over portion of the catchment. The present study is confined  
to rectangular shaped catchments which eliminates this possibility.  
It may also occur that there is an initial retention loss in which case  
the storm duration should exceed the theoretical concentration time  
of the basin for maximum peak runoff. This possibility will be examined  
later. In that case the concentration time is measured from the time  
runoff commences. Whether or not initial abstraction takes place, it  
is possible to solve for design storm duration  $t_d$  and peak runoff  $i_{ep}$   
from the equation for concentration time and the storm intensity re-  
lationship such as (4.37). The total depth of loss is designated s,  
in the same units as ti, where i is the rainfall rate and t is time.  
Subscript i refers to initial loss and u to uniform loss rate in time.

Case I - Uniform loss rate

For a uniform loss rate over the entire storm duration  $i_e = i - f$  (4.39)  $= i - s_u/t_d$  (4.40) In general if

$$i = \frac{a}{(b+t_d)}c$$
(4.37)

and  $t_c = (L/zi_e^{n-1})^{1/n}$  (4.19)

then by substitution of (4.37) into (4.39) and (4.39) into (4.19), one obtains an implicit expression for the design storm duration  $t_d$ :

$$t_{d} = \left[\frac{L}{z\left\{\frac{a}{(b+t_{d})}c - f\right\}^{n-1}}\right]^{1/n}$$
(4.41)

If the Manning-Strickler stage-discharge equation is employed and c is taken as unity,

$$t_{d} = \frac{(Lk^{\frac{1}{6}}/7.7\sqrt{Sg})}{(\frac{a}{b+t_{d}} - \frac{s_{u}}{t_{d}})^{\frac{2}{5}}}$$
(4.42)

$$= \frac{F}{\left(\frac{1}{b+t_{d}} - \frac{U}{t_{d}}\right)^{\frac{2}{5}}}$$
(4.43)

where F = {  $\frac{Lk^{1/6}}{7.7\sqrt{Sg} a^{2/3}}$  } 3/5 (4.44)

is defined as the catchment retardation factor and  

$$U = s_{\mu}/a$$
 (4.45)

is defined as the infiltration factor. Equ. (4.43) cannot be solved explicitly for  $t_d$  or for the peak runoff rate so the equation was solved for F as a function of  $t_c$  and U for various values of b. The results are summarized in Fig. 4.10, from which the concentration time may be read knowing the catchment characteristics, namely length L, absolute roughness k, slope S, storm characteristic a and uniform infiltration loss  $s_u$ . Unless the storm duration is known, it may be difficult to assess  $s_u$ . In many cases the infiltration rate  $f = s_u/t_d$  is known instead of the total volume lost, and the dashed lines on Fig. 4.10 may therefore be of more use in estimating concentration times. Once f and  $t_d$  are established,  $s_u$  may be evaluated. The maximum storm runoff rate may now be evaluated from the equation

$$i_{ep} = \frac{a}{b + t_d} - \frac{s_u}{t_d}$$
(4.46)

which is plotted in Fig. 4.12 in dimensionless terms with  $t_d = t_c$  evaluated from Fig. 4.10. Subscript e refers to excess rain and p to that corresponding to peak runoff rate. It will be observed that the

maximum rate of runoff per unit area does not occur for small smooth basins, except for no losses. For real losses represented by U there is some basin configuration represented by F which results in a higher rate of runoff per unit area. This is because for any U the rate of loss reduces with increasing F and hence increasing  $t_c$ , and this effect predominates over the lower storm intensity. On the other hand for short storms, the rate of loss would have to be high to produce a certain U, hence the rate of runoff is affected.



Fig. 4.10 Design storm duration for uniform losses, b = 0.4 h

### Case II - Initial loss

If all the storm input is initially absorbed or taken up in filling depression storage, runoff will not commence until the storage is full If the storage or loss volume is  $s_i$  per unit area, then the time until runoff commences is

$$t_{i} = s_{i}/i$$
For peak runoff,  

$$t_{c} = t_{d} - t_{i}$$

$$= \frac{(Lk/7.7\sqrt{Sg})^{\frac{3}{5}}}{(\frac{a}{b + t_{d}})^{\frac{2}{5}}}$$
(4.47)
(4.47)
(4.47)

Therefore  $t_d = F(b+t_d)^{\frac{1}{5}} + I(b+t_d)$  (4.49) where I is the initial retention factor,  $s_i/a$ .

This equation was solved for F for various  $t_d$  and I and the results are plotted in Fig. 4.11 for various values of I. It will be noted that the resulting storm durations for peak runoff are invariably greater for initial losses than for uniform losses.

It will be observed from Fig. 4.12 however, that the peak runoff rate is higher for initial loss than for uniform loss. For no loss both theories yield identical results as would be expected while for increasing losses the results diverge. The peak runoff per unit area for case II, however, occurs for the smallest, smoothest and steepest catchment.

For losses comprising a combination of initial storage and uniform infiltration, Fig. 4.12 may be interpolated, taking note that each line for a particular loss function is drawn assuming the other type of loss is zero.

## Surface losses

The losses to be deducted from precipitation include interception on vegetation and roofs, evapotranspiration, depression storage and infiltration. The remaining losses may be divided into initial retention and a time-dependent infiltration.

The loss function is really a function of many variables, including antecedent moisture conditions and ground cover. Infiltration is timedependent and an exponential decay curve was proposed by Horton (1939), Holton (1961), and others. The infiltration typically reduces from an initial rate of about 50 mm/h down to 10 mm/h over a period of about an hour. The rates, especially the terminal loss rate, will be higher for coarse sands than for clays.



Fig. 4.11 Design storm duration for initial losses, b = 0.4 h

The time-decaying loss rate could be approximated by an initial loss plus a uniform loss over the duration of the storm. Values of initial and uniform losses used in the United States are tabulated in Table 4.<sup>-</sup> The mean uniform loss rates are averages for storms of 30 minute duration, and the initial losses include the initial 10 minute rapid infiltration or saturation amount.



Fig. 4.12 General peak runoff, b = 0.4 h

TABLE 4.1 Initial and uniform loss rates

	SURFACE LOSSES				
	Initial 10	oss - (mm)	Uniform infiltration rate - (mm/h)		
	Surface Retention	Infiltration			
Paved	up to 1		0		
Clay	î'' 5	20	2 – 5		
Loam	'' 5	30	5-15		
Sandy soil	'' 5	40	15-25		
Dense vege- tation	" 12	-			

In the case of ploughed lands, and other especially absorptive surfaces an additional initial loss of up to 10 mm or more may be included. Allowance must also be made for reduced losses from covered areas (paved or roofed). The values should be used with caution until more appropriate data are available.

## Roughness

The Manning-Strickler drag equation is dimensionally homogeneous. By including the absolute roughness k as a variable, it loses the empiricism of the Manning equation. In fact the drag effect is very insensitive to the roughness, as it is to the power of 1/6. Thus any inaccuracy in selecting k is masked by the equation. It is preferable to overestimate k as the drag equation tends to predict too rapid flow concentration unless this is done. This is due to the tortuosity of the flow path over rough surfaces. In fact the original form of the Manning equation and the Strickler approximation for the roughness were never intended for overland flow where the depth of flow is comparable with the roughness and Reynolds numbers are of the order of 1 000. Table 4.2 may be used as a guide for surface roughness k.

TABLE 4	. 2	Surface	roughness
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ABSOLUTE ROUGHNESS,	k (mm)
Concrete lined storm drains Concrete paving Gravel Lawn, turf Weeds Ploughed land Boulders and rubble Dense vegetation 1	0.5 1 5 20 50 150 500 000+

The length of drainage path and slope influence the concentration time more than the roughness. Runoff follows a circuitous path over natural land and the ground slope along the flow path is therefore flatter than the net slope. A similar lag occurs with runoff from roofs (2 to 5 minutes lag). Allowance should be made for these effects in establishing the retardation factor F. EXAMPLE:

The example illustrates the use of the design charts for determining design storm duration, peak runoff and the effect of canalization. The use of consistent units in the equations should be noted.

Calculate the peak 20-year runoff from a catchment which is 500 m wide and 2 000 m long with a uniform longitudinal slope of 1/500, and an effective absolute roughness of 10 mm. The infiltration rate is 20 mm per hour. Neglect overland flow time for the purposes of the example. The 20 year storm factor 'a' for the station is 90 mm and the time factor 'b' is 0.4 h.

The retardation factor is

$$F = \left[\frac{2\ 0.00\ x\ 0.01}{7.7\sqrt{9.8/500}\ 0.09^{\frac{2}{3}}}\right]^{\frac{1}{6}} = 151\ s^{0.6}$$

Uniform retention factor for total loss  $U/t_d = 20/90 = 0.222$  per h. Interpolating between the lines on Fig. 4.10 storm duration = 2.2 h, therefore U = 0.48 and from Fig. 4.12  $i_{ep}/a = 0.16/h$ . The peak excess runoff is therefore  $i_{ep} = 0.16 \times 90 = 14.4$  mm/h. The peak rate of runoff is 14.4 x 500 x 2 000/3 600 x 1 000 = 4.0 m<sup>3</sup>/s. The value of 'C' in the rational formula is

$$\frac{{}^{1}\text{ep}}{{}^{1}\text{ep} + s/t_{d}} = \frac{14.4 \text{ mm/h}}{14.4 + 20} = 0.42$$

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