CHAPTER 10

## FLOW IN CIRCULAR DRAINS

ADVANTAGES OF PIPES

Pipes are in many ways a very convenient means of removing stormwater. They are buried so that they are unobtrusive, they are structurally strong, and the hydraulic properties of circular pipes are favourable in comparison with other types of closed conduits.

In regions sewered many years ago and where storm runoff is relatively low, wastewaters and storm drainage are transported in the same pipes. In those situations closed conduits were essential for health. This is not always done nowadays, although surplus capacity is often allowed in sewers for stormwater which may enter the system at gulleys or leaking manholes or joints. Waste sewers are rarely designed to run full, whereas stormwater drains are. The hydraulic grade line ideally runs along the soffit of stormwater drains at design flow so that manholes are not surcharged.

Although the design of pressure pipes is beyond the scope of this section, basic principles of hydraulics of circular pipes are presented together with some design rules.

HEAD LOSS IN FULL PIPES

The energy of a flowing fluid expressed per unit weight of fluid, is termed the head. It comprises elevation head, pressure head and velocity head. In accordance with Bernoullis's equation for an ideal fluid the total energy at one section is equal to that at another section:
$z_{1}+\frac{p_{1}}{W}+\frac{v_{1}{ }^{2}}{2 g}=z_{2}+\frac{p_{2}}{W}+\frac{v_{2}{ }^{2}}{2 g}$
where $v=$ mean velocity across a section
$\frac{v^{2}}{2 g}=$ velocity head (units of length)
$g=$ gravitational acceleration
$\mathrm{P}=$ pressure
$p / w=$ pressure head (units of length)
$w=$ unit weight of fluid
$z=$ elevation above selected datum
If there occurs head loss due to friction and turbulence between sections 1 and 2 , then the term $h_{f}$ (head loss) should be added to the
right hand side of (10.1). Strictly the velocity head should be multiplied by a coefficient to account for the variation in velocity across the section of the conduit. The average value of the coefficient for turbulent flow is 1.06 and for laminar flow it is 2.0 . For the Bernoulli equation to apply the flow should be steady, i.e. there should be no change in velocity with time at any point. The flow is assumed to be one-dimensional and irrotational and the fluid should be incompressible.

The respective total energy head and hydraulic grade line are illustrated in Fig. 10.1 (Stephenson, 1979). For most practical cases the velocity head is small compared with the other components, and it may be neglected.


Fig. 10.1 Heads along a pipeline

The throughput or capacity of a pipe of fixed dimensions is controlled by the total head difference between the ends. This head is consumed by friction and other turbulence losses. The losses at bends, junctions, changes in diameter and at manholes (sudden expansions) are usually less than the friction loss. Gravity or free flowing pipelines are laid to the friction gradient, with additional allowances at changes in section. The head loss at such a section is a fraction of the velocity head;
$h_{L}=K_{1} v_{1}{ }^{2} / 2 \mathrm{~g}$
where $K_{1}=\left(1-A_{1} / A_{2}\right)^{2}$
for a sudden expansion from area $A_{1}$ to $A_{2}$.

TABLE 10.1 Nikuradse Roughness for Pipe Materials
Value of $k$ in $m m$ for new, clean surface unless otherwise stated.

| Finish: | Smooth | Average | Rough |
| :--- | :--- | :--- | :--- |
| Glass, drawn metals | 0 | 0.003 | 0.006 |
| Steel, PVC or A C | 0.015 | 0.03 | 0.06 |
| Coated steel vitrified clay | 0.03 | 0.06 | 0.15 |
| Galvanized, ver cement lined | 0.06 | 0.15 | 0.3 |
| Cast iron or | 0.15 | 0.3 | 0.6 |
| Spun concrete or wood stave | 0.3 | 0.6 | 1.5 |
| Riveted steel | 1.5 | 3 | 6 |
| Foul sewers, tuberculated water mains | 6 | 15 | 30 |
| Unlined rock, earth | 60 | 150 | 300 |

TABLE 10.2 Hazen-Williams Friction Coefficients C

| Type of Pipe | New | $\begin{aligned} & 25 \text { years } \\ & \text { old } \end{aligned}$ | Condition |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\begin{aligned} & 50 \text { years } \\ & \text { old } \end{aligned}$ | Badly corroded |
| PVC | 150 | 140 | 140 | 130 |
| Smooth concrete, AC | 150 | 130 | 120 | 100 |
| Steel, bitumen 1 ined |  |  |  |  |
| galvanized | 150 | 130 | 100 | 60 |
| Cast iron | 130 | 110 | 90 | 50 |
| Riveted steel |  |  |  |  |
| vitrified, woodstave | 120 |  | 80 | 45 |

FRICTION EQUATIONS
Darcy-Weisbach equation

A number of empirical relationships for friction head loss in terms of pipe diameter and discharge were developed for specific use in water works practice. These equations (such as that of Hazen-Williams) were applicable within their sphere of development but cannot be extrapolated heedlessly. Following research by Reynolds, van Karman and others into turbulance, boundary layer theory was developed to yield a flow-head loss relationship for a range of flows in pipes. The Darcy-Weisbach friction equation is one equation resulting from this research: $S=\lambda v^{2} / 2 g D$

This equation together with the associated Moody diagram (Fig. 10.2) or the Colebrook-White equation for the friction factor $\lambda$ (or $f$ in USA practice) is slowly gaining acceptance as the most rational method for estimating friction head losses in pipes. S is the head loss gradient, and $D$ is the pipe internal diameter. For non-circular conduits or partly


Fig. 10.2 Friction factors as a function of Reynolds number and relative roughness
full conduits $D$ may be replaced by $4 R$, where $R$ is the hydraulic radius A/P, A is the cross-sectional area of flow and $P$ is the wetted perimeter

The Darcy friction factor is yielded by the Colebrook-White equation (1939) :
$\frac{1}{\sqrt{\lambda}}=-2 \log \left(\frac{k}{3.7 \mathrm{D}}+\frac{2.5}{\mathrm{R}^{\sqrt{\lambda}}}\right)$
In view of the complicated relationship between the Darcy friction factor $\lambda$, Reynolds number $R_{e}$ and the relative roughness $k / D$, explicit head loss charts have been prepared (Ackers, 1969; Watson, 1979). Such a chart is given as fig. 10.3. $k$ is a measure of the boundary roughness, termed the Nikuradse roughness (see Table 10.1). The Reynolds number is $R_{e}=v D / v$ or $4 v R / v$ for part-full pipes
where $v$ is the kinematic viscosity of the fluid

Hazen-Williams equation

Despite its sound background and apparent simplicity the Darcy-Weisbach equation does not directly yield discharge rate for any given pipe and head loss gradient except in the turbulent rough zone of the Moody diagram. Equations such as that of Hazen-Williams and Manning remain in use because they can be solved directly for discharge rate. The bounds of applicability of these equations requires clarification. The equations also appear in specific units, and a dimensionless rendering would be welcome. Diskin (1960) presented a useful comparison of the friction factors from the Hazen-Williams and Darcy Equations.

The Hazen-Williams equation is widely used in water engineering practice. The equation is
$v=K_{W} C_{W} D^{0.63} S^{0.54}$
where $K_{w}$ is 0.354 in metre units and 0.550 in foot units. $C_{w}$ is the Hazen-Williams coefficient (see Table 10.2). The Hazen-Williams equation may be rewritten in the following dimensionless form:
$v=0.044 C_{w}\left(R_{e} / C_{w}\right)^{0.075} \sqrt{g D S}$
The Hazen-Williams coefficient $C_{w}$ is a function of $\lambda$ and $R_{e}$ and values may be plotted on a Moody diagram (see Fig. 10.2). It will be observed from Fig. 10.2 that lines for constant Hazen-Williams coefficient coincide with the Colebrook-White lines only in the transition zone. In the completely turbulent zone for non-smooth pipes the coefficient will actually reduce the greater the Reynolds number. The Hazen-Williams equation should therefore be used with caution for high Reynolds numbers and rough pipes.


Fig. 10.3 Friction loss chart for pipes flowing full, $k=0.3 \mathrm{~mm}$

Manning equation

The Manning equation is widely used for head losses in open channel flow calculations and for part full pipes.
The equation is
$v=\frac{K_{n}}{N} R^{2 / 3} S^{1 / 2}$
where $K_{n}$ is 1.0 in $S$.I. (metre) units and 1.486 in ft units.
Strickler proposed the following relationship between the Manning coefficient $N$ and the effective size of boundary roughness $k$ :
$N=0.13 \mathrm{~K}_{\mathrm{n}} \mathrm{k}^{1 / 6} / \sqrt{\mathrm{g}}$ (10.11)
thus $v=7.7(R / k) \quad 1 / 5 \sqrt{S R g}$ (all units)
i.e. the Manning coefficient depends only on the boundary roughness, $k$.

The Manning equation is therefore only applicable to turbulent flow with a rough boundary. It is however easier to use than the Darcy equation and has thus retained popularity despite its limitations. Typical values of $N$ are given in Table 10.3 and a relationship between $N$ and $k$ is given in Table 10.4. In fact Strickler's equation is analogous to the Darcy equation with a simpler $1 / 6$ th power equation for $\lambda$ instead of the Colebrook-White $\log$ equation.

TABLE 10.3 Manning's $N$

| Smooth glass | 0.010 |
| :--- | :--- |
| Concrete, galvanized or lined steel | 0.011 |
| Cast iron | 0.012 |
| Slimy or greasy sewers | 0.013 |
| Rivetted steel, vitrified | 0.015 |
| Rough concrete | 0.018 |

TABLE 10.4 Relationship between Manning Coefficient $N$ and roughness $k$. $\left(n=0.13 K_{n} k^{1 / 6} / \sqrt{g}\right)$

| N | $\mathrm{k}(\mathrm{m})$ | $\mathrm{k}(\mathrm{ft})$ |
| :--- | :--- | :--- |
| 0.01 | 0.0002 | 0.0006 |
| 0.012 | 0.0006 | 0.0019 |
| 0.015 | 0.0022 | 0.0072 |
| 0.02 | 0.012 | 0.039 |
| 0.025 | 0.048 | 0.156 |
| 0.03 | 0.142 | 0.466 |
| 0.04 | 0.80 | 2.625 |
| 0.05 | 3.05 | 10.00 |

NON-CIRCULAR CROSS SECTIONS
A circular pipe is normally the most economic if it is to be designed to resist internal pressures. A circular shape has the shortest circumference per unit of cross sectional area, consequently it requires least wall material, as well as being easy to manufacture.

Ellipical or horseshoe shapes are often adopted for sewers or drains. They have different strength and hydraulic characteristics to circular pipes. Vertical elliptical pipes (major axis vertical) have smaller wetted perimeters when running partly full with low flows, consequently the velocity is higher than for a circular pipe, which assists in flushing. The vertical load on a vertical elliptical pipe is less than on a circular pipe with the same cross sectional area, and the strength is greater because the curvature is sharper at the top.

Horizontal elliptical pipes (major axis horizontal) are sometimes used where vertical loads are low or clearance is limited. Running partly full they will discharge relatively high flows at small depths of flow which may be an advantage if head is limited.

Arch shapes with flat bottoms have similar hydraulic characteristics to horizontal elliptical shapes for low flow under partly full conditions. The arch shape is usually the most practical shape in tunnelling.

Provided the cross-sectional shape does not differ much from circular i.e. it could be elliptical or even rectangular, the Darcy equation is applicable. $4 R$ is substituted for $D$ in the equation and in the Reynolds number.

UNIFORM FLOW IN PART-FULL CIRCULAR PIPES

Most friction formulae for full pipe flow have been used for part full flow. For a circular pipe of diameter $D$ running at depth $y$, the cross-sectional area of flow is:
$A=\frac{D}{4}^{2} \cos ^{-1}\left(1-\frac{2 y}{D}\right)-\left(\frac{D}{2}-y\right) \sqrt{y D-y^{2}}$

The wetted perimeter is:
$P=D) \cos ^{-1}\left(1-\frac{2 y}{D}\right)$
Using these equations charts may be prepared yielding a dimensionless relationship between flow depth and cross sectional area and hydraulic radius as a proportion of the full depth value, i.e. $A / A_{f}$ and $R / R_{f}$ versus y/D, as given as Fig. 10.4.

Also indicated on the chart are lines indicating the velocity ratio $v / v_{f}$ and the discharge ratio $Q / Q_{f}$ versus $y / D$ for uniform flow. For nonuniform flow the ratio of friction gradient $S_{f} / S$ versus $y / D$ is indicated assuming $Q / Q_{f}=1$. These lines are dependent on the assumed friction loss equation. If Manning's equation is used with roughness $N$ independent of depth, then the resulting relationships are as indicated. In

fact if Strickler's approximation for N is used, then N is independent of flow depth and dependent only on boundary roughness. If the DarcyWeisbach equation is adopted, then assuming a constant friction factor $\lambda$ i.e. independent of depth of flow, similar relationships could be plotted. The friction factor $\lambda$ is known to vary with Reynolds number though, especially for shallow depths and correspondingly low Reynolds numbers. In such cases the relationships between $v / v_{f}, ~ Q / Q_{f}$ and $y / D$ are not unique unless a varying $\lambda$ is used i.e. $\lambda$ is a function of two variables, $k / R$ and Reynolds number, so a different line will apply for each case.

Camp (1946) performed tests to determine the variation of $N$ and $\lambda$ with depth. His charts are presented by ASCE (1969), but it should be borne in mind those relationships are not completely in accordance with the Colebrook-White equation for the reasons indicated above.

Using Figs. 10.3 and 10.4, given any three of the five variables Q, D, S, v and $y$, the other two may be determined. The flow conditions for full-bore flow ( $y / D=1$ ) are yielded simultaneously.Designate $Q_{f}=$ flow at full bore and $v_{f}=v e l o c i t y$ at full bore. Now assume the flow, pipe diameter and slope ( $Q, D$ and $S$ ) are known, and $y / D$ and $v$ are to be determined. Read $Q_{f}$ and $v_{f}$ from Fig. 10.3 and using the ratio $Q / Q_{f}$, read $y / D$ from Fig. 10.4. Hence also read $v / v_{f}$ from Fig. 10.4 and calculate $v$ knowing $\mathrm{v}_{\mathrm{f}}$.

As another example, given $Q=50 \ell / \mathrm{s}, \mathrm{S}=0.0005$ and $\mathrm{y} / \mathrm{D}=0.25$, find the necessary diameter $D$ and corresponding velocity: From Fig. 10.4, $Q / Q_{f}=0.135$ so $Q_{f}=370 \mathrm{e} / \mathrm{s}$ and from Fig. $10.3 \mathrm{D}=525 \mathrm{~mm}$ and $v_{f}=1.7$ $\mathrm{m} / \mathrm{s}$. Now from Fig. $10.4, \mathrm{v} / \mathrm{v}_{\mathrm{f}}=0.7$ hence $\mathrm{v}=1.2 \mathrm{~m} / \mathrm{s}$.

An interesting fact is illustrated in Fig. 10.4. The flow for a partly full pipe is greater than the flow through a fully charged pipe if the depth of flow is between $82 \%$ and $100 \%$ of the diameter. The reason for this is that the wetter perimeter increases rapidly but the area does not, as the pipe fills up over the last portion. The additional capacity shculd not be relied upon however as the slightest irregularity may cause the pipe to run full.

CRITICAL DEPTH AND HYDRAULIC JUMPS IN PIPES

Bernoulli's energy equation applies to the flow in circular drains running full or part full. Thus for no friction or energy losses, $z+y+\frac{v^{2}}{2} g=$ constant
or
$z+y+Q^{2} / 2 g^{2}=$ constant
where $z$ is bed elevation, $y$ is water depth and $v$ is mean velocity. In accordance with this equation, the specific energy is a minimum at some depth $y_{c}$ termed the critical depth. It is derived in implicit form by differentiating the energy equation with respect to depth and setting the differential equal to zero. Then
$Q_{c}=\left(g_{c}{ }^{3} / B_{c}\right)^{1 / 2}$
or in dimensionless numbers
$\frac{Q_{c}}{\left(g D^{5}\right)^{1 / 2}}=\frac{\left(A_{c} / D^{2}\right)^{3 / 2}}{\left(B_{c} / D\right)^{1 / 2}}$
Both area $A$ and surface width $B$ are functions of flow depth $y$. Thus
$A / D^{2}=\frac{1}{4} \cos ^{-1}(1-2 y / D)-\left(\frac{1}{2}-y / D\right)\left(y / D-y^{2} / D^{2}\right)^{1 / 2}$
(10.19)
and $B / D=2\left(y / D-y^{2} / D^{2}\right)^{1 / 2}$

TABLE 10.5 Flows for Varying Values of Critical Depth in Circular Channels

| $y / D$ | $\mathrm{A} / \mathrm{D}^{2}$ | B / D | $\overline{\mathrm{y}} / \mathrm{D}$ | $Q_{c} /\left(g D^{5}\right)^{\frac{1}{2}}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.00 | 0.0000 | 0.0000 | 0 | $0.000{ }^{-3}$ |
| 0.05 | 0.0147 | 0.4359 | 0.017 | $2.699 \times 10^{-3}$ |
| 0.10 | 0.0409 | 0.6000 | 0.039 | 10.68 |
| 0.15 | 0.0739 | 0.7141 | 0.061 | 23.77 .. |
| 0.20 | 0.1118 | 0.8000 | 0.082 | 41.80 |
| 0.25 | 0.1535 | 0.8660 | 0.103 | 64.64 |
| 0.30 | 0.1982 | 0.9165 | 0.122 | 92.18 |
| 0.35 | 0.2450 | 0.9539 | 0.146 | 124.16 |
| 0.40 | 0.2934 | 0.9798 | 0.168 | 160.55 |
| 0.45 | 0.3428 | 0.9950 | 0.190 | 201.18 |
| 0.50 | 0.3927 | 1.0000 | 0.212 | 246.11 |
| 0.55 | 0.4426 | 0.9950 | 0.236 | 295.17 |
| 0.60 | 0.4920 | 0.9798 | 0.260 | 348.63 |
| 0.65 | 0.5404 | 0.9539 | 0.284 | 406.76 |
| 0.70 | 0.5872 | 0.9165 | 0.310 | 470.00 |
| 0.75 | 0.6318 | 0.8660 | 0.336 | 539.68 |
| 0.80 | 0.6736 | 0.8000 | 0.364 | 618.09 .. |
| 0.85 | 0.7115 | 0.7141 | 0.393 | 710.22 .. |
| 0.90 | 0.7445 | 0.6000 | 0.424 | 829.29 .. |
| 0.95 | 0.7707 | 0.4359 | 0.459 | $1001.8 \times 10^{-3}$ |
| 1.00 | 0.7854 | 0.0000 | 0.500 | $\infty$ |

Employing these relationships values of $A / D^{2}, B / D$ and the corresponding value of $Q_{C} /\left(g^{5}\right)^{1 / 2}$ are given in Table 10.5 as functions of y/D.

Diskin (1958 and 1962) indicates that the following experimental equation fits the relationship between $y_{C} / D$ and $Q /\left(g D^{5}\right)^{1 / 2}$ for $y / D$ between 0.05 and 0.85 $y_{c} / D=\left\{1.05 Q /\left(g D^{5}\right)^{1 / 2}\right\}^{0.533}$

The minimum specific energy corresponding to critical depth is (Jenker, 1962)
$\mathrm{H}_{\text {min }}=\mathrm{Y}_{\mathrm{c}}+\mathrm{A}_{\mathrm{c}} / 2 \mathrm{~B}_{\mathrm{c}}$
This may be evaluated from the functions of $A / D^{2}$ and $B / D$ versus $y / D$ previously tabulated. In general whether the depth is critical or not, the relationship between depth and specific energy can be plotted in non-dimensional form (Fig. 10.5), from
$\frac{E}{D}=\frac{Y}{D}+\frac{Q^{2} / g D^{5}}{2\left(A / D^{2}\right)^{2}}$
It should be noted that for high specific energy the pipe may be surcharged in the subcritical condition in which case the term $y / D$ represents the depth plus pressure head. In this case the lines above $y / D=1$ in Fig. 10.5 extend at 45 degrees above the soffit of the pipe.

The specific momentum of the flow may likewise be evaluated in dimensionless terms. The total momentum per unit weight of water is
$M=\frac{Q^{2}}{g A}+A \bar{y}$
or in dimensionless terms
$\frac{M}{D^{3}}=\frac{Q^{2}}{g D^{5}} \frac{D^{2}}{A}+\frac{A}{D^{2}} \frac{\bar{y}}{D}$
where $\bar{y}$ is the depth from the top water surface to the centroid of the section. $\bar{y} / D$ as a function of $y / D$ was evaluated numerically and is give in Table 10.5. It may be shown (Henderson, 1966, p 84) that
$A \vec{y}=\frac{D}{24}^{3}\left(3 \sin \frac{\theta}{2}-\sin { }^{3} \frac{\theta}{2}-3 \frac{\theta}{2} \cos \frac{\theta}{2}\right)$
where $\cos \theta=1-2 y / D$

The dimensionless specific momentum function is plotted in Fig. 10.6. In many cases the sequent depth should exceed the diameter of the conduit. In this case the conduit will run full and pressurized. The corresponding specific momentum is
$\frac{M}{D^{3}}=\frac{Q^{2}}{g D^{5}} \frac{4}{\pi}+\frac{\pi}{4} \frac{\bar{y}}{\bar{D}}$
where $\bar{y}$ is the pressure head above the centre-1ine of the conduit. A limited range of values is plotted in Fig. 10.6.

A hydraulic jump in a conduit can entrain air which causes additional complications. The air is likely to be released from solution and small bubbles will rise to the top of the pipe, creating a part-full pressurized flow situation. Whether the air is in the form of bubbles or a pocket along the soffit of the conduit, the head losses will be higher than for pure water flowing at the specified rate. The volume occupied by entrapped air can be as great as $25 \%$ for low pressure systems (Mussalli, 1978).



The rate of air entrainment of a free hydraulic jump in a rectangular channel was determined from model experiments by Kalinske and Robertson (1943). Recent tests on jumps in pipes indicate this equation underpredicts the air entrainment rate. An equation of the following form is indicated
$Q_{a} / Q_{w}=0.03(F-1)$
where $Q_{a}$ is the volumetric air entrainment rate and $F$ is the upstream Froude number $\left(Q^{2} B / \mathrm{gA}^{3}\right)^{1 / 2}$. Wisner et al (1975) also advocate the removal of air by hydraulic means.

The total volume of discharge immediately downstream of the jump before the air has had time to dissolve, is therefore $Q+Q_{a}$ and the friction loss is a function of this total flow. Air may subsequently be released at manholes in gravity lines or by air valves in pressure lines. In fact air is a nuisance in many cases as it results in head losses which restrict the capacity of the pipe.


Fig. 10.7 Circular pipe part full

## FLOOD ROUTING

Although the kinematic method of routing, ignoring dynamic terms, is sufficiently accurate for most drains (Stephenson, 1980), for large drains the full hydrodynamic equations can be employed for greater accuracy. Harris (1970) found the characteristic method expensive on computer time and preferred a progressive average lag method. In many cases the kinematic equations enable routing to be performed easily. The cross sectional area of flow in a circular conduit running part full (see Fig. 10.7) may be written as
$A=\frac{D}{4}^{2}\left(\frac{\theta}{2}-\cos \frac{\theta}{2} \sin \frac{\theta}{2}\right)$
and $\mathrm{P}=\mathrm{D} \frac{\theta}{2}$
Thus if wo take the angle $\theta$ subtended at the centre as the variable the continuity equation becomes
$\frac{D^{2}}{8}\left(1+\sin ^{2} \frac{\theta}{2}-\cos ^{2} \frac{\theta}{2}\right) \frac{\partial \theta}{\partial t}+\frac{\partial Q}{\partial x}=q$
where $q$ is the inflow per unit length and $Q$ is the total flow rate in the pipe. This may be solved for $\theta_{2}$ after a time interval $\Delta t$ in finite difference form
$\theta_{2}=\theta_{1}+\left(q-\frac{\Delta \theta}{\Delta x}\right) \frac{8 \Delta t}{1)^{2}\left(1+\sin ^{2} \frac{\theta}{2}-\cos ^{2} \frac{\theta}{2}\right)}$
and in terms of the new $\theta$, using the Strickler approximation for friction losses,
$Q=\frac{7.7 \sqrt{S g}}{k^{1 / 6}} \frac{D^{2}}{4}\left(\frac{\theta}{2}-\cos \frac{\theta}{2} \sin \frac{\theta}{2}\right)\left\{\frac{D}{4}\left(1-\frac{\cos \frac{\theta}{2} \sin \frac{\theta}{2}}{\theta / 2}\right)\right\}$
(10.33) and ( 10.34 ) are solved successively at different points along the drain at successive time intervals to yield a history of $Q$ versus $x$ and $t$.

## Approximate Method

The following assumptions may often be made in the case of a slug of water released into a drain (Stephenson, 1977).

1. There is no base flow in the drain.
2. A volume of water $U$, i.e. ${ }^{\prime}$ 'slug' is injected into the drain over
a length $L$, at uniform depth $y$. The cross-sectional area of flow is approximately $2 / 3$ y $B$ (which is not accurate unless y is small), where $B$ is the surface width.
3. The volume of water travels down the drain at an average velocity $v$ where $\lambda$ is the Darcy friction factor, $R$ is the hydraulic radius and $S$ is the bedslope.


Fig. 10.8 Attenuation of surges in circular drains


Fig. 10.9 Storm drain profile (vertically exaggerated)
4. The extremities of the volume of water $U$ are travelling outwards relative to the average velocity by the celerity $c=\sqrt{g y}$, i.e. the two ends of the water are travelling at $v-c$ and $v+c$ respectively down the pipe.
5. The depth of water remains uniform over the length.

Then employing the continuity equation and an approximation for area of flow it may be shown
$\frac{\Delta y}{D} \doteqdot-0.77\left(\frac{y}{D}\right)^{2} \sqrt{\frac{Y}{D}-\left(\frac{y}{D}\right)^{2}} \sqrt{\frac{f}{S}} \frac{D^{2} \Delta x}{Q}$
(10.35) was solved in steps to yield a relationship between $y$ and $x$ as shown in Fig. 10.8. This chart represents the attenuation in depth of the surge as it travels down the drain. The chart albeit only an approximation will be satisfactory as a design aid for many problems e.g. the estimation of the attenuation of flow in stormwater drains after high intensity, short duration storms. Thus by routing a storm down a drain it will reduce in intensity, enabling drain sizes to be minimized. It is not suggested that the drain diameter be reduced along the length, as the storm could presumably occur anywhere along its length. However, in summating inputs along the length, the inputs from higher up in the catchment could be suitably routed using the chart so that the total capacity is less than the sum of the inputs. It may also be employed to estimate the amount of water needed to flush a sewer (Watson, 1937).

To estimate the initial flow depth of any input if the duration and total inflow are known one may use the approximation
for $y \ll d, y \div\left(\frac{\lambda U^{2}}{10 g D S t^{2}}\right)^{1 / 4}$
(10.36)

BACKWATERING AND GRADUALLY VARIED FLOW

The computation of backwater surface profiles in circular channels can be done by the standard step method (Henderson, 1966). Nalluri and Tominson (1978) presented a direct step method necessitating the use of tables.

Backwatering is easily executed by hand with the assistance of a graph such as Fig. 10.4 depicting proportional area of flow, hydraulic radius and energy gradient.

## PROGRAM FOR BACKWATERING IN PART-FULL PIPES

Using the equations for the geometric properties for partly full circular pipes one is able to backwater in a circular pipe. It is a simple matter to program the equations. Such a program is appended.

The friction equation employed in the program is that of Manning, with a constant ' $N$ ' value. The Manning friction equation is rendered independent of whether metres or feet are used by expressing it as
$V=g^{1 / 3} R^{2 / 3} S^{1 / 2} / 2 \cdot 14 N$
(10.37)

Data is read in via device 9 (see lines 3 and 6 in the program). Free format is used and data is read in the following order:
First line : N, Q, D, C, G, Y(1), E
Second and subsequent lines: (N such lines): Z(M), X(M), T(M)
where $N=$ number of cross sections considered
$\mathrm{Q}=\mathrm{flow}$ rate
D = pipe diameter
C = Manning's coefficient
( $)=$ gravitational acceleration
$Y=$ water depth at section 1 .
$\mathrm{I}=$ permitted error in depth during computations
$Z(M)=$ invert level of pipe at section $M$ measured above any constant datum
$X(M)=$ distance to next cross section. For the last pipe this may be set at zero
$T(M)=$ turbulent loss coefficient in the pipe immediately before the cross section $M\left(\Delta H=\mathrm{TV}^{2} / 2 \mathrm{G}\right)$

The printout on device 5 (see lines 39, 41 and 44) includes input data as well as water levels and velocities at each section. The program can be used to backwater upstream in the case of subcritical flow or downstream in the case of supercritical flow. In the latter case, pipe lengths should be input as negative values.

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L-0ccos
.CCO2
L.0004 2C
L.0005
L.000S
L.0007 11
L=000E
L.0C05
L.001C 15
L.0011 2C
L.0012
L.00112
L.0015
L.0016
L.0016
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L.0020
L.0021
L.0022
L.0023 60
L.0024
L.0025
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L.0027
L.0028
L.0025
L.0030 70
L.0031
L.0053 65
L.0034
L.0035
L.003\epsilon 50
L.0037
L.0038 100
L.0035
L.0040 110
L.OC4Z 
L.0044 150
1.0045 120
L.0045
L.OC4E
- CCO
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C
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C
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2C
11
19
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GACKHATEF IN CIFC FIFES

READ 9,10 )N, $Q, O, C, G, Y(1), E$
FCFMAT( 7 Y$)$
DC $100 \mathrm{M}=1, N$
DC $100 \mathrm{M}=1, N$
$\operatorname{READ}(G, 11) \angle(N), X(N), T(N)$
$\left.\begin{array}{l}\text { READ } \\ \text { FCFMAT } \\ \text { (11) } \\ 3\end{array}\right)$
IF(M.LE.I)GCTO 15
$I F(M=L E(1) G$
$Y(M)=Y(N-1)$
$Y(M)$
$K=0$
$N(N)=Z(N)+Y(N)$
$T H=2$ **ARCOS (.9999*(1.-2.*Y(M) 10))

$V=Q / A$
$H E=w(M)+\vee * v / 2 \cdot / G$
$\mathrm{P}=0 *$ * 5 * T
$P=0 * * S$
$R=A / P$
$\mathrm{B}=2 . * \mathrm{D} * \mathrm{SCFT}(Y(\mathrm{M}) / \mathrm{D}-\mathrm{Y}(\mathrm{M}) * * 2 . / ワ / \mathrm{D})$

IF(M.LE•1)GOTO E5
IF $(M \cdot L E-1) G O T O E E$
$S A=(S(M-1)+S(M))$
$H F=H(N-1)+S A * X(N-1)+T(N) * v * v / 2 \cdot / G$
IF (ABS(HF-HE).LE.E)GO TO $\rightarrow 0$
IF (ABS (HF-HE). LE
$F S=2 * 0 * E / G / A *: 3$.
OY=(HF-HE)/(1•-FS+3.*S(M)*X(M-1)/R/2.)
YY $=(H F-M E) /(1$
$Y(M)=Y(M)+D Y$
$Y(M)=Y(M)+D Y$
$I F(Y(M): L E=D) G O$ TO 70
$Y(M)=D$
GC TJ 90
GC $\quad$ K
$K=K+1$
IFRK.GE EIGO TO :10
GETO20
$H(1)=Z(i)+V * V / Z . / G+Y(:)$
U $\{1\}=V$
$G C$
$G C T O 100$
$H(M)=H F$
$U(M)=V$
CONTINUE
inue



STCP
END
DINENSI(A $\gamma(30), 2(90), \bar{x}(50), T(90), U(90), w(90), 5(50), 4(50)$
STGP

