CHAPTER 11

DRAINAGE NETWORK OPTIMIZATION

INTRODUCTION

One of the engineer's objectives is to produce a satisfactory design such that the overall cost is a minimum. This is referred to as cost optimization.

Economic pressures and the advent of electronic computers have both prompted many researchers to search for cost saving design methods. The conventional design approach is based on a set of design standards and criteria, with no alternatives compared. Optimization is in practice often approximated by investigating a series of designs. The designer selects on the basis of his best professional judgement a system layout and a combination of pipe diameters and grades. He will design and estimate the cost of a number of alternative layout-size-slope combinations. Using conventional design methods it is not feasible to design and evaluate more than a few alternatives. Computers enable a larger number of alternatives to be evaluated, but an optimal solution is not guaranteed.

An ideal optimization model would produce the minimum cost design by simultaneously varying both network layout and pipe design. This has been attempted by a number of researchers. The fact remains, however, that no method is in existence for obtaining such an overall solution. There are, however, design methods which produce the minimum cost design for a system of a given network layout. i.e. if flows are known, the system becomes 'linear' and can be optimized directly.

THE VARIABLES

A drainage network for any area can be anything from a single pipe to an interconnected tree-like network, (Fig.11.1) depending on the size and configuration of the catchment, and the relative economic advantages and cost of the drainage system.

The designer of a drainage network must decide the following:

- Layout plan of the township or catchment
- Location of drains within the permitted zones
- Spacing of inlets
- Location of manholes at bends and changes in grade and diameter.

Capacity of inlets
Size of surface gulleys
Diameters of subsurface drains
Gradient of drains
Design details of inlets, manholes, drops, branches, etc.

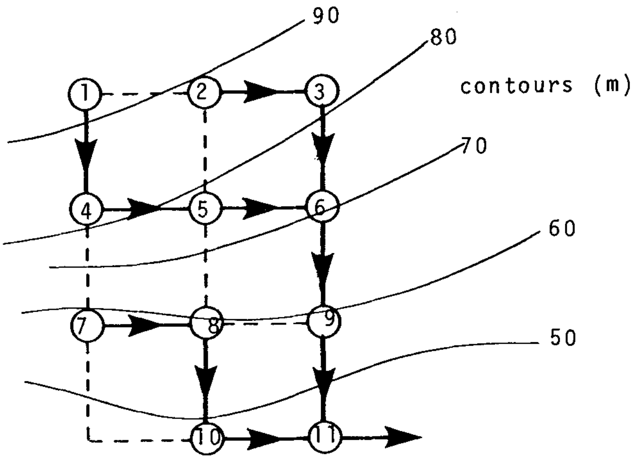

Fig. 11.1 Drainage network with alternative layouts

Many of the design parameters can be selected independently, thus eliminating the number of alternatives. Drain layouts are frequently fixed by the road layout. Drains are located on the downhill side of roads or on both sides. Surface gulley sizes depend on the design flows and drain inlet spacing. Assuming each inlet accepts the total gulley flow at that point, the problem is to select gulley capacity and sub-surface drain capacity for least total cost. The problem is analogous to the Highway Drainage System discussed later in the chapter. In the case of systems with fixed layouts and consequently known design flows, the problem is to determine the optimum combination of diameter and grade for each link pipe (Fig. 11.2)

Costs of drainage components depend primarily on the flow in the conduit. This may be a variable if the network layout and inlet spacing are to be determined, or may be fixed if the layout and inlet spacing are pre-selected. Drainage layouts are more often than not of the tree-like layout, with the flow in each successive branch being cumulative. There is often no problem in selecting between alternative routes as

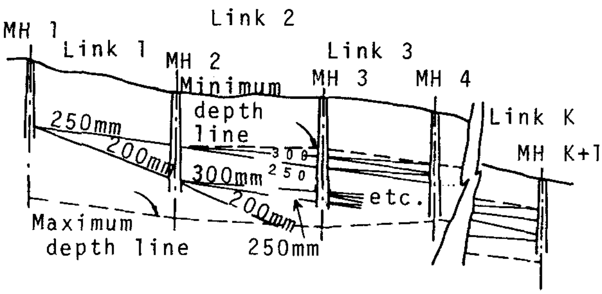

Fig.11.2 Drain profile showing solution procedure for dynamic programming optimization.

the most economic layout is usually with drains flowing downhill. This is unlike water supply networks where the flow in each pipe and the layout have many possibilities (one of which is optimal) (Stephenson, 1979). Cost of drains depend on:

Diameter

Depth, which influences excavation and pipe wall thickness

Manhole spacing

Locality, such as in built-up or open ground

Dajani and Hasit (1974) and Merritt and Bogan (1973) present some typical cost data, but this is highly dependent on locality and cost escalation, so the engineer is urged to compile cost data to suit the particular project.

There will be a number of practical constraints on the system to be optimized. These include:

Permissible pipe depth depending on ground conditions

Available pipe strengths

Minimum grade or velocity to avoid deposition

Maximum grade to avoid erosion or noxious gas release

Minimum diameter for access purposes

Maximum surcharge

Pipe diameters must be those commercially available

Gradient and diameter must be consistent with flow rate and friction factor

Invert levels of successive pipes at intersections (manholes) must be equal or represent a fall

Head losses at inlets and branches must be allowed for.

Whereas there are many mathematical methods for the selection of a least-cost system, some of the methods cannot accommodate all the constraints rigorously. In particular linear programming and geometric programming require continuous variables. Mixed integer programming can be used to select discrete values, as can dynamic programming.

Implicit in all the following techniques is that the runoff rate into each drain or per unit area of catchment, is known. This means the design storm intensity and consequently the design storm duration, have been estimated beforehand. Flow rates are used as input to the analyses. The computations subsequently produce pipe sizes and grades, and correspondingly, flow velocities. In order to determine the concentration time of the system, the flow should strictly be routed through the system. This may indicate a new concentration time which should equal design storm duration, upon which it would be necessary to revise precipitation rates in the light of the known intensity - duration relationships. In fact the situation is even more complicated as the concentration time and consequently design storm vary down the system. A design approach allowing all possible degrees of freedom would be prohibitively costly though.

It is also assumed the pipes will run full at design flow in the following sections. This could be varied though to suit the design standards.

DYNAMIC PROGRAMMING FOR OPTIMIZING COMPOUND PIPES

One of the simplest optimization techniques, and indeed one which can normally be used without recourse to computers, is dynamic programming, e.g. Meredith, 1971; Walsh and Brown, 1973; Kally, 1980. It is in fact only a systematic way of selecting an optimum program from a series of events and does not involve any mathematics. The technique may be used to select the most economic diameters of a compound pipe which may vary in diameter along the length depending on grades and flows. For instance consider a drain fed by a number of inlets. The diameter of the main is increased as input takes place along the line. The problem is to select the most economic diameter for each section of pipe.

A simple example demonstrates the use of the technique; Consider the line in Fig. 11.3. Two inlets feed stormwater to a drain, and the hydraulic gradient, assuming pipes flow full, should not be above ground level. The elevations of each point and the lengths of each

TABLE 11.1 Dynamic programming optimization of a compound drain

I

HEAD AT B	HYDR. GRAD.	DIA. mm	COST
H_B	h_{A-B}	D_{A-B}	COST $
13	.009	250	50000
18	.0065	260	52000
23	.004	300	60000

II

H_C =	7			12			17		
H_B	h_{B-C}	D_{B-C}	COST $	h_{B-C}	D_{B-C}	COST $	h_{B-C}	D_{B-C}	COST $
13	.006	310	31000 50000 81000	.001	430	43000 50000 93000	-	-	-
18	.011	270	27000 52000 79000*	.006	310	31000 52000 83000*	.001	430	43000 52000 95000
23	.016	250	25000 60000 85000	.011	270	27000 60000 85000	.006	310	31000 60000 91000*

III

H_C	h_{C-D}	D_{C-D}	COST $
7	.001	430	86000 79000 165000
12	.0035	340	68000 83000 151000*
17	.006	310	62000 91000 153000

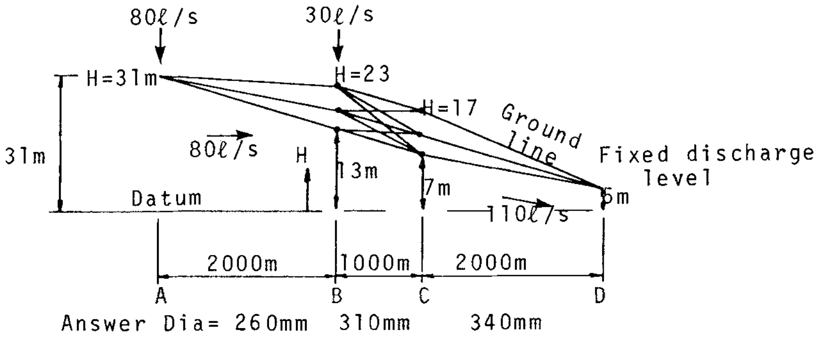

Fig. 11.3 Profile of drainpipe optimized by dynamic programming

section of pipe are indicated. The cost of pipe is 10 cents per millimetre per metre length of pipe. The analysis will be started at the upstream end of the pipe (point A). The most economic arrangement will be with minimum cover (zero say), at point A. The depth at point B may be anything between 13 m and 31 m above the datum, but to simplify the analysis, we will only consider three possible heads with 5 m increments between them at points B and C.

The diameter of the pipe between A and B, corresponding to each of the three allowed heads may be determined from a head loss chart and is indicated in Table 11.1 (I) along with the corresponding cost.

The number of possible hydraulic grade lines between B and C is 3 x 3 = 9, but one of these is an adverse gradient so may be disregarded. In Table 11.1 (II) a set of figures is presented for each possible hydraulic grade line between B and C. Thus if H_B = 23 and H_C = 17 m then the hydraulic gradient from B to C is 0.006 and the diameter required for a flow of 110 ℓ/s is 310 mm. The cost of this pipeline would be 0.1 x 310 x 1 000 = $31 000. Now to this cost must be added the cost of the pipe between A and B, in this case $60 000 (from Table 11.1 (I)). For each possible head H_C there is one minimum total cost of pipe between A and C, marked with an asterisk. It is this cost and the corresponding diameters only which need be recalled when proceeding to the next section of pipe. In this example, the next section between C and D is the last and there is only one possible head at D, namely the discharge level.

In Table 11.1 (III) the hydraulic gradients and corresponding diameters and costs for Section C - D are indicated. To the costs of pipe for this section are added the costs of the optimum pipe arrangement up to C. This is done for each possible level at C, and the least total cost selected from Table 11.1 (III). Thus the minimum possible total cost is $151 000 and the most economic diameters are 260, 310 and 340 mm for Sections A-B, B-C and C-D respectively. It may be desirable to keep pipes to standard diameters in which case the nearest larger standard diameter could be selected for each section as the calculations proceed or each length could be made up of two sections; one with the next larger standard diameter and one with the next smaller standard diameter, but with the same total head loss as the theoretical result. Alternatively one could select head intervals to result in real diameters.

It will be seen that the technique of dynamic programming reduces the number of possibilities to be considered by selecting the least-cost arrangement at each step. Of course many more sections of pipe could be considered and the accuracy would be increased by considering more possible heads at each section. A computer may prove useful if many possibilities are to be considered, and there are standard dynamic programming programs available.

OTHER APPLICATIONS

Dynamic programming has been used to select an optimum layout of a drainage system and the optimum sizes and profiles. It becomes cumbersome to optimize both the layout and profiles especially as the number of legs increases since large computer capacity is required. The required core storage in the computer increases with the square of the number of variables for this type of optimization.

Merrit and Bogan (1973) presented a program for selection of least-cost profile and pipe sizes for a sewer (or drainage system). It was necessary to select a layout and inlet positions on a plan manually. Restraints on pipe minimum and maximum depths and feasible pipe diameters were possible. The program commences analysis at the top of each leg, knowing the flow, and works down each stage between inlets, considering various pipe diameters and grades. Each successive leg depends on the invert level of the end of the previous upstream pipe or pipes. Drops and pumping stations are permitted.

Two computer models were developed - one essentially for sewers where the maximum and minimum velocities could be specified and depths of flow were indicated by the program, and one model which assumed the pipes to flow full but not surcharged. The latter model is the one applicable to storm water drains.

Argaman et all (1973) developed a mathematical model, also using dynamic programming, to optimize the layout of drains or sewers. A number of arbitrary layouts are set out, and flow directions pre-defined. The computer then selects which routes can be omitted, and calculates diameters and grades. In order to reduce the number of variables the pipes were assumed to be parallel to the ground surface, (alternatively the pipe grades could be pre-selected). The depth of flow could be limited, and a range of flows could be considered. This proves useful for sewers where low flow conditions are important. Steady-state flow was assumed and head losses at junctions were neglected. A cost function (a polynomial) is used to estimate pipe costs for different diameters and depths. If the permissible pipe diameters were pre-chosen, the optimized cost of the system as indicated by the program was found to be considerably higher than the cost of a system where any diameter was possible (i.e. a continuous function was used).

Dajani and Hasit (1974) used linear programming to select pipe sizes in a drainage layout. It was necessary to linearize the Manning head-loss equation which somewhat limits the validity of the program. Linear programming produces a least-cost system only if all relationships between variables are linear i.e. of the form $x = ay + b$.

Separable programming methods were employed by Dajani et al (1972) while discrete dynamic programming was used by Mays and Yen (1975) to optimize a fixed layout. Other approaches are illustrated by Barlow (1972) and Davis (1975).

HIGHWAY DRAINAGE SYSTEMS

The layout and spacing of drains along a dual lane carriageway can be difficult to design. The system comprises carriageway drains, cross drains, carrier drains and outfalls such as in Fig. 11.4. Runoff from

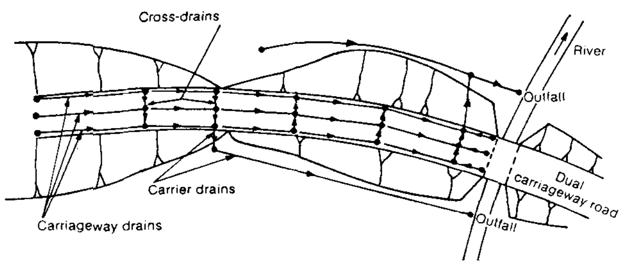

Fig. 11.4 Typical highway storm drainage network.

the pavement enters the carriageway drains in the median strips and centre island via gulleys or through filter media. Cross drains connect the carriageway drains at intervals which in turn lead to the carrier drains. Manholes are placed at maximum intervals, at intersections, changes in pipe diameter, alignment or slope for clearing and maintenance.

In the case of highway drains, the scope of layouts is limited and to some extent repetitive. If the layout is fixed, the only variables are drain size and slope, and previously discussed methods are possible.

The additional variables which were studied by Templeman and Walters (1979) are manhole spacing and cross-drain positions. They proposed a dynamic programming solution. The following section describes their method of solution.

It is assumed that the inflow to each pipe is known. Thus the relationship between concentration time and design storm intensity is not considered. Fig. 11.5 shows the system for which manhole spacing, pipe diameter and grades are to be optimized.

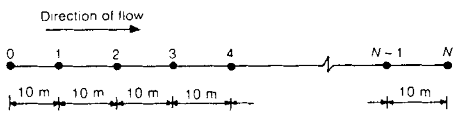

Fig. 11.5 Possible manhole locations along a pipe run.

A drain between fixed manholes 0 and M (flowing towards N) is to have an unknown number of manholes at intervals which are multiples of 10 m for simplicity. Only 3 possible diameters will be considered.

Now if a manhole is positioned at 1 it can be connected to manhole 0 only by the link 0-1. Let there be 10 discrete depths at each manhole and three pipe diameters for each pipe. Although the top pipe depth would normally be fixed at a minimum, let there be 10 possible depths at 0. Then there will be a set of 10 minimum costs for link 0-1 as for the dynamic programming selection of pipe diameters and grades. If a manhole is positioned at 2, it can be connected directly to 0 by link 0-2 or via 1 with link 0-1-2. For each discrete depth at 2, a minimum cost design can be found by selecting the cheaper of 0-2 or 0-1-2. Only the additional cost of link 1-2 need be calculated now as minimum cost designs for link 0-1 have already been calculated and stored.

For a manhole at position 3, possible links are 0-3, 0-1-3, 0-2-3 and 0-1-2-3. For each discrete depth at 3 only one of these four configurations must produce a least cost design. As cheapest designs for drains up to 2 have already been calculated only the costs of 0-3, 1-3 and 2-3 need be calculated at this stage.

The process is continued to manhole N. At this point 30 cheapest designs will be available (one for each of ten pipe depths and three pipe sizes at N). The lowest cost must be selected. Now it remains to trace back through the system to locate manhole locations and pipe sizes and slopes for the optimal system for any given depth constraint.

The method can readily be extended to include positioning of cross drains (or inlets in the case of a combination of gulleys and drains) (Fig. 11.6). Again it must be assumed that the intensity of runoff is known i.e. is not a function of flow path.

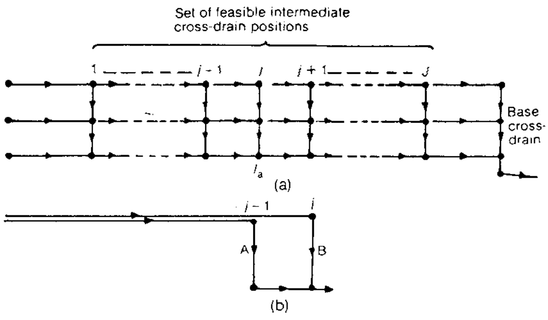

Fig. 11.6 (a) Discrete positions of cross drains
(b) Flow paths with assumed equal travel times

The procedure is similar to that above. One proceeds down the drain making decisions at each possible cross-drain position. The design and cost downstream of any position depend on the depth at that position and decisions downstream. Assuming the costs up to the previous position have been determined, then for each cross-drain position links from all feasible upstream positions must be determined. The cheapest solution resulting in any discrete depth at the manhole in question is stored. When the end manhole is reached, select the cheapest alternative and trace back to determine the least cost system.

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