CHAPTER 13

HYDRAULICS OF BRIDGES

INTRODUCTION

Bridges, culverts, causeways and fords are constructed by engineers to get traffic across waterways. A single span over a full channel width would not interfere with the flow in the channel. Economics and structural limitations usually require the bridge length to be less than the water surface width at maximum flow. The restriction on width and opening height often has the effect of backing up the water upstream of the bridge. The backwater thus created floods additional land upstream. A compromise between bridge opening and flooded area can often be achieved on an economic basis.

The problem of flooding over the top of the bridge and the consequent hazard to traffic is difficult to assess, and for this reason engineers correctly tend to be more cautious than pure economic grounds would indicate necessary. Like with the design of culverts, good hydraulic design of the approach to the bridge can minimize the backwater effect. The bridge embankments and piers can be shaped to streamline the flow.

In the case of bridges, the control is usually at the entrance to the channel constriction, so streamlining the approach flow can increase the hydraulic capacity of the opening.

The local reduction in width will cause higher water velocities than average, with the result that the scour regime is affected and local scour in the vicinity of the bridge is likely unless some form of bed and bank protection is employed. (Laursen, 1962; Kindsvater, 1957).

Beyond the constriction, the flow expands again to the full cross section of the channel. The flow expands after the constriction at a rate of 5° to 6° from the centre line on each side. There is dead water on the downstream side of the embankments, and even some circulation which dissipates energy. The energy loss through the constriction is a function of the velocity head difference in the constriction and downstream. The downstream water level, in the case of subcritical flow, is controlled further downstream. It may be normal level if there is a long uniform channel downstream. Fig. 13.1 shows the flow pattern between two encroaching embankments.

The U.S. Department of Transport (1978) has conducted considerable research into the hydraulics of flow through bridges. The research took place in the form of model tests and examination of field data. The

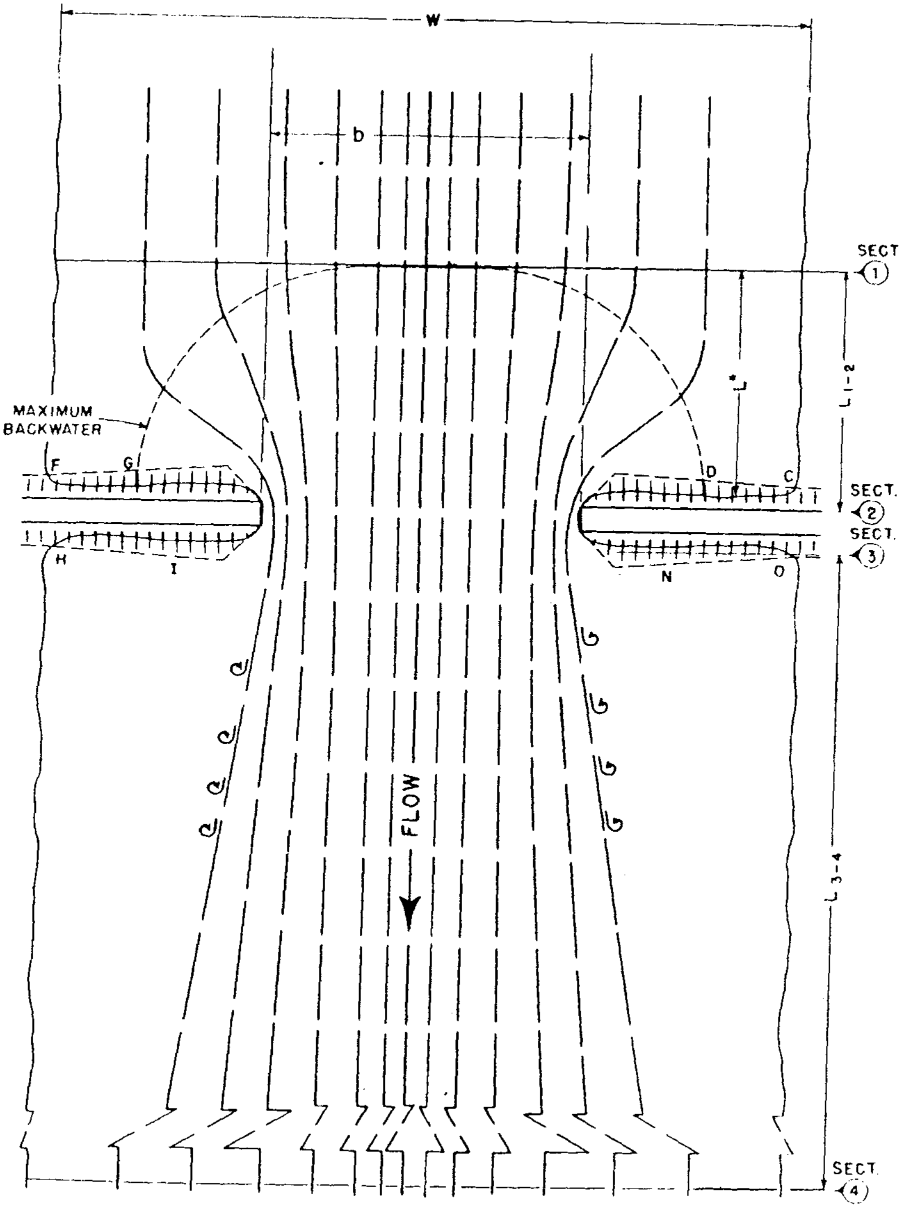

Fig. 13.1 Flow lines for normal bridge crossing

results of the research culminated in the way of design curves for estimation of head losses and water surfaces through bridges. Much of the following is based on their work.

FLOW THROUGH GAP

The velocity through the trapezoidal shape formed by two facing embankments across a river gap obeys a relationship of the form

$v = C\sqrt{2g(y_1 - y_2)}$ (13.1)

and flow $Q = C\,A_2\sqrt{2g(y_1 - y_2)}$ (13.2)

where the depth y_1 is upstream of the gap and y_2 is in the gap as indicated in Fig. 13.2. In the case of drowned flow y_2 is taken as the depth downstream or y_3 above bed level in the gap. The value of the coefficient C was found by Naylor (1976) to vary from 0.75 to 1.09 with a mean of 0.9. In the case of supercritical flow through the gap y_2 should be replaced by the critical depth y_c.

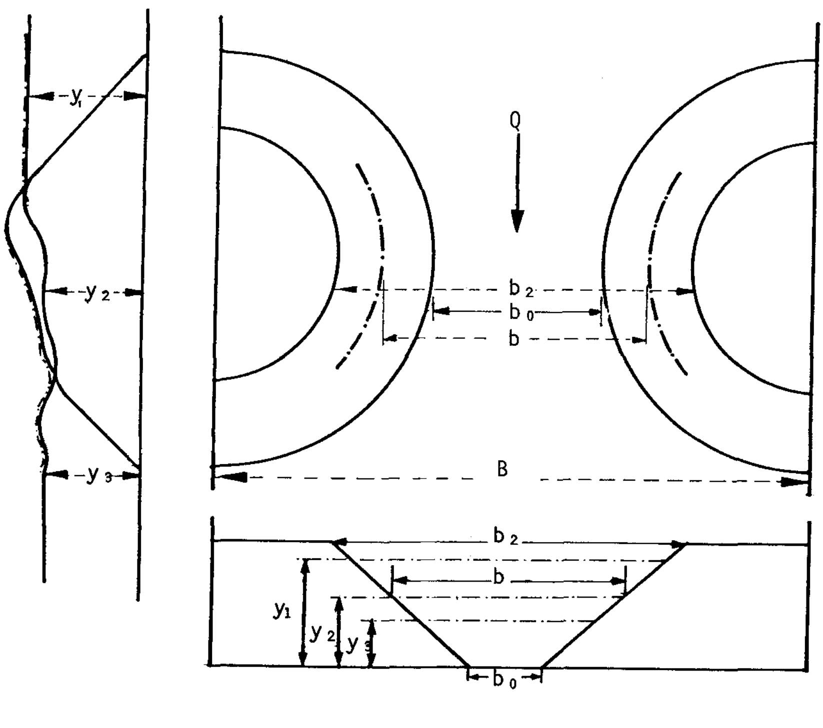

Fig. 13.2 End-tipped embankment

The critical depth in a trapezoidal section is given by

$$y_c/y_1 = 0.4\left[1-1.5p + \sqrt{1 + 2p + 2.25p^2}\right] \qquad (13.3)$$

where $p = b_o \tan \Theta/2y_1$ and b_o is the bottom width at the control section in the gap. The value of y_c/y_1 varies between 0.67 for wide gaps and 0.80 for triangular gaps. Thus for a triangular gap with free flow,

$$Q = 0.26\sqrt{2g}\ y_1^{5/2}/\tan \Theta \qquad (13.4)$$

For the triangular shaped gap it is possible to solve for the inside slope for stability of a granular or rockfill surfacing. Eliminating v and y_1 from Equs. 13.1 and 13.4 and combining with an expression for stable stone size d, we get an expression for Θ in terms of d and Q (Stephenson, 1979),

$$\frac{(\tan \Theta)^{2/5}}{\cos \Theta\sqrt{\tan^2 \phi - \tan^2 \Theta}} \approx \frac{8.25\ dg^{1/5}(S-1)}{Q^{2/5}} \qquad (13.5)$$

The equation may be solved by trial and error or iterative techniques.

SURFACE PROFILE

There may be three types of flow through a bridge waterway. The corresponding water surface profiles are depicted in Fig. 13.3 and described below:

(I) If the water surface is above critical depth at every section the flow is subcritical (type I flow). This is the condition normally encountered in practice and the calculation procedures following generally refer to this type of flow.

(II) The flow depth may pass through critical in the constriction. Under these conditions the water depth upstream becomes independent of downstream conditions. If the depth passes through critical in the constriction, but not below critical depth downstream, it is referred to as type IIA flow. If the flow depth drops below downstream critical depth, it is referred to as type IIB flow. In this case a hydraulic jump will occur below the constriction if downstream depth is above critical depth.

(III) If normal flow in the channel is supercritical, the water level in the constriction will rise as illustrated in Fig. 13.3. Undulations of the water surface will probably occur and waves may occur upstream and downstream. No backwater in the normal sense will occur.

The backwater effect due to a constriction in a channel may be evaluated from energy considerations. The analysis hereunder is for the case of a straight channel sloping uniformly with the bridge perpendicular

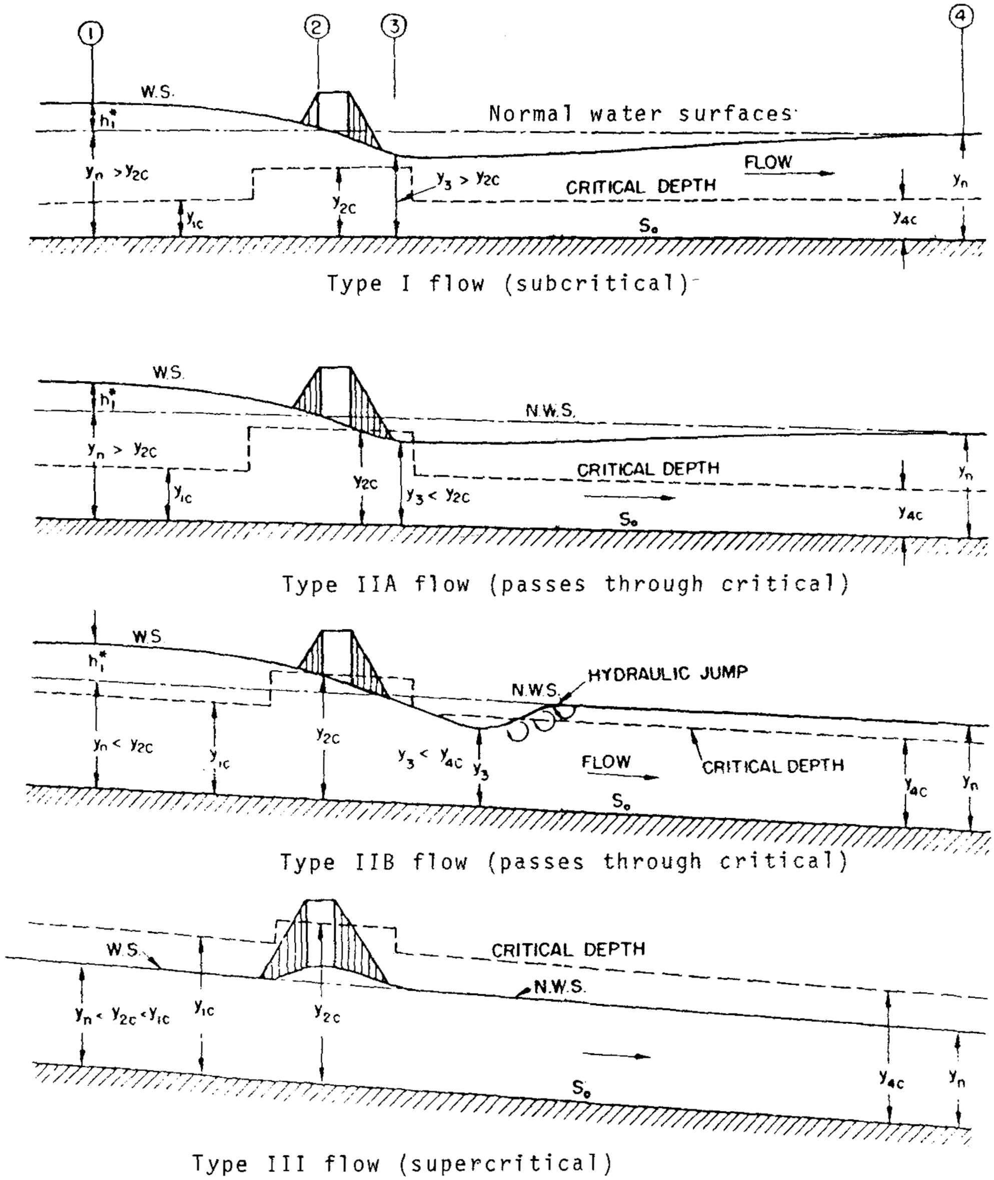

Fig. 13.3 Flow profiles past embankments

to the flow direction. Flow is assumed to be steady and subcritical. Without the bridge constriction energy loss in friction would just balance the drop in bed level between sections 1 and 4 in Fig. 13.3. The additional head loss due to the constriction will be designated h_b and this is assumed to be given by an expression of the form

$$h_b = K \alpha_2 v_2^2/2g \tag{13.6}$$

where v_2 is the average velocity at cross section 2 (the constriction) for water level at normal depth for the river section and α is a velocity energy coefficient, yielded by an integration across the section of qv:

$$\alpha = \frac{\sum_i q_i v_i^2}{Q V^2} \tag{13.7}$$

Q is the total discharge and V is the mean velocity across the section

$$\text{Thus } h_b = y_1 - y_4 = \frac{\alpha_4 v_4^2}{2g} - \frac{\alpha_1 v_1^2}{2g} + \frac{K \alpha_2 v_2^2}{2g} \tag{13.8}$$

Now since sections 1 and 4 are essentially the same, $\alpha_1 = \alpha_4$ and by continuity $A_1 v = A_2 v_2 = A_4 v_4$. Therefore

$$h_b = \frac{K\alpha_2 v_2^2}{2g} + \alpha_1 \{ (\frac{A_2}{A_4})^2 - (\frac{A_2}{A_1})^2 \} \frac{V_2^2}{2g} \tag{13.9}$$

It should be noted that $y_1 - y_4$ is not the difference in water levels; it represents the buildup in water level (or backwater) upstream of the bridge. In addition there will be friction head losses due to normal flow.

The backwater head loss coefficient K_b for flow normal to a symmetrical restriction may be read from Fig. 13.4. Here M is the bridge opening ratio,

$$Q_b/(Q_a+Q_b+Q_c) \tag{13.10}$$

where Q_b is flow which would pass through the same section as the bridge opening without the bridge there (See Fig. 13.1).

Since A_1 is not known until h_b has been determined, it is necessary to estimate h_b initially from

$$h_b = K\alpha_2 v_2^2/2g \tag{13.11}$$

The value of A in (13.9) can then be determined.

The backwater head loss is also affected by:

(i) The number, size, shape and orientation of piers in the constriction.

(ii) The eccentricity of the bridge in the river section

(iii) Skewness of the bridge relative to the direction of the river.

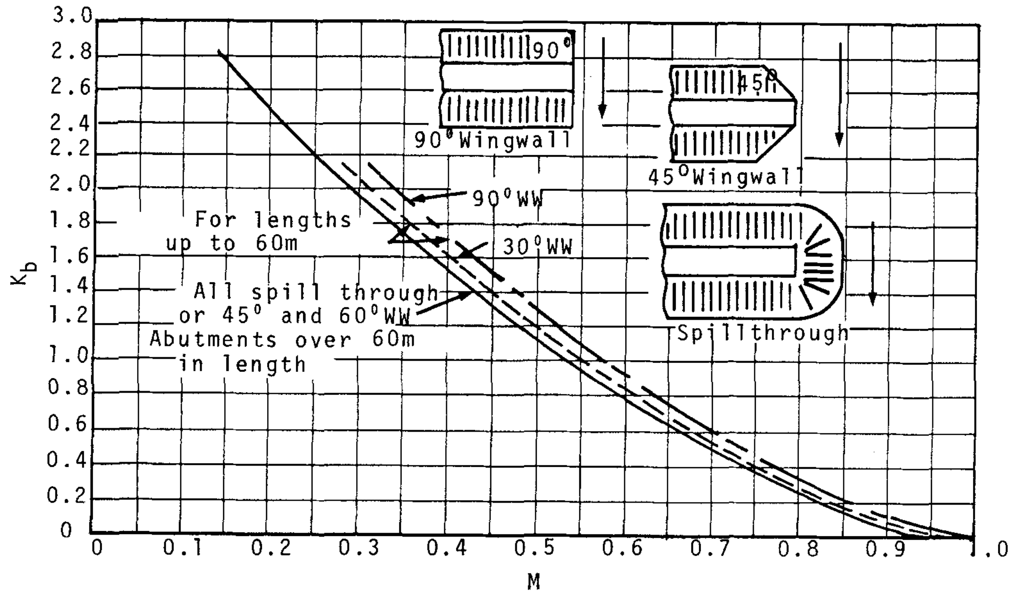

Fig.13.4 Backwater coefficient base curves (subcritical flow)

The influence of these effects on K is given in Figs. 13.5 to 13.7. Thus the K to use should be $K = K_b + \Delta K_p + \Delta K_e + \Delta K_s$ (13.12)

In the case of skew openings, the projected width of opening and not the total width of opening should be employed to determine M.

It should be noted that the results here are based on the assumption of one-dimensional flow. Laursen (1970) indicates that lateral flow can significantly increase the backwater effect.

DROP IN WATER LEVEL

The difference between the water level upstream and downstream of the bridge embankment is not the same as the backwater. The water level in the restriction is difficult to evaluate theoretically and it was investigated by model testing. Fig. 13.8 presents the resulting data. To use the curve, compute the contraction ratio M and read off D_b the differential level ratio where $D_b = h_b/(h_b + h_3)$. Now with the previously computed backwater for a normal crossing, h_b, compute h_3 the drop in level.

$$h_3 = h_b \left(\frac{1}{D_b} - 1\right) \qquad (13.13)$$

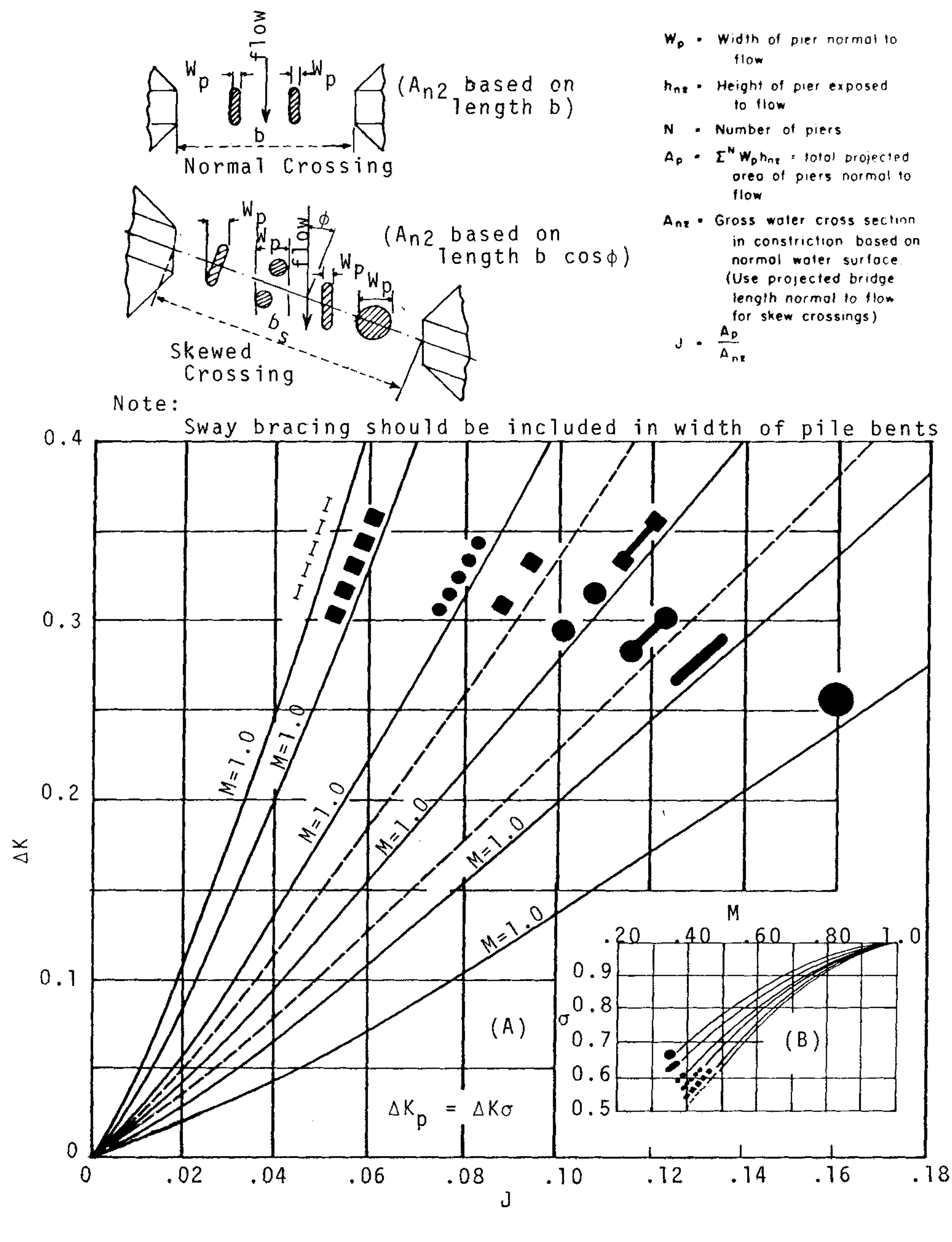

Fig. 13.5 Increment on backwater coefficient for piers

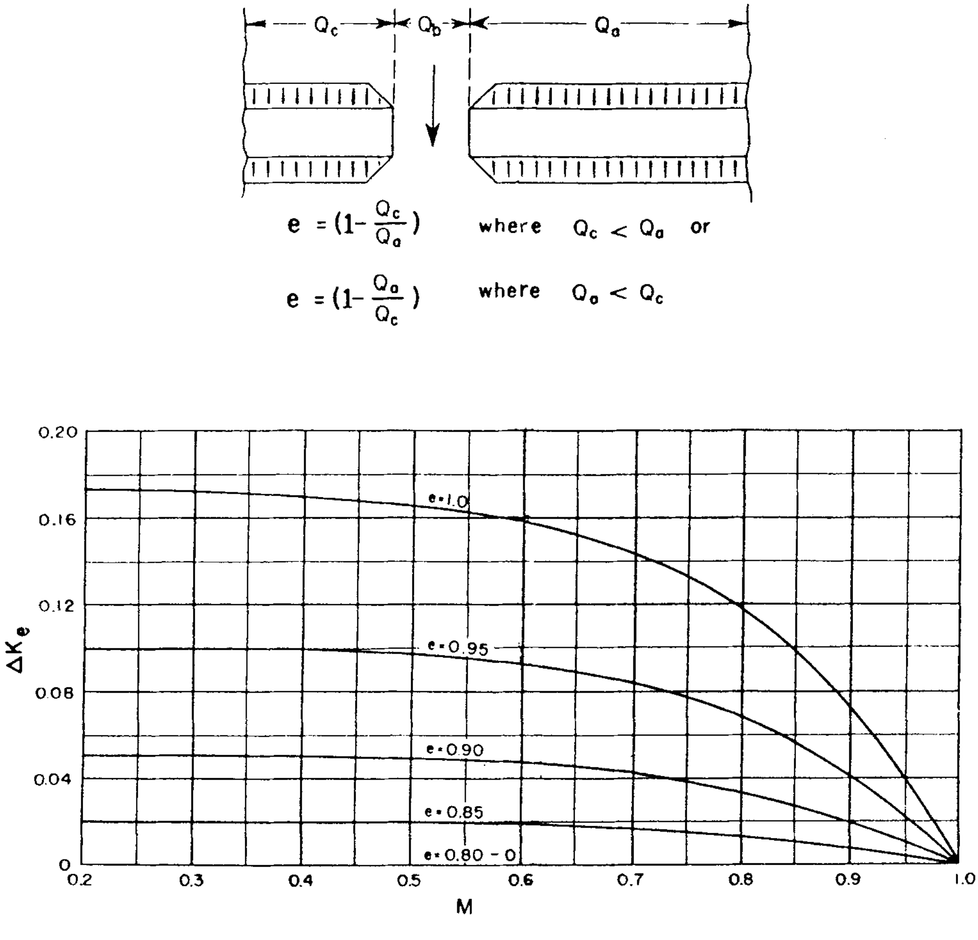

Fig. 13.6 Increment on backwater coefficient for eccentricity

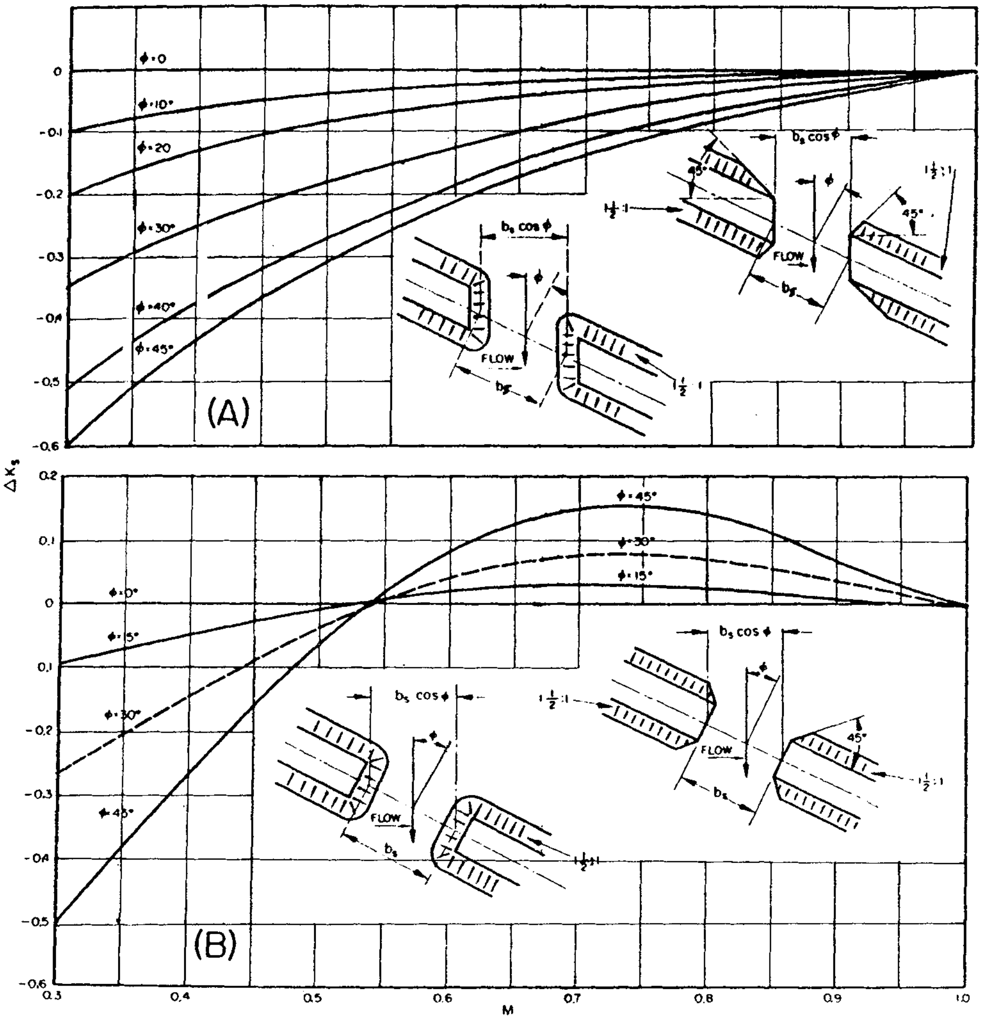

Fig. 13.7 Increment on backwater coefficient for skew

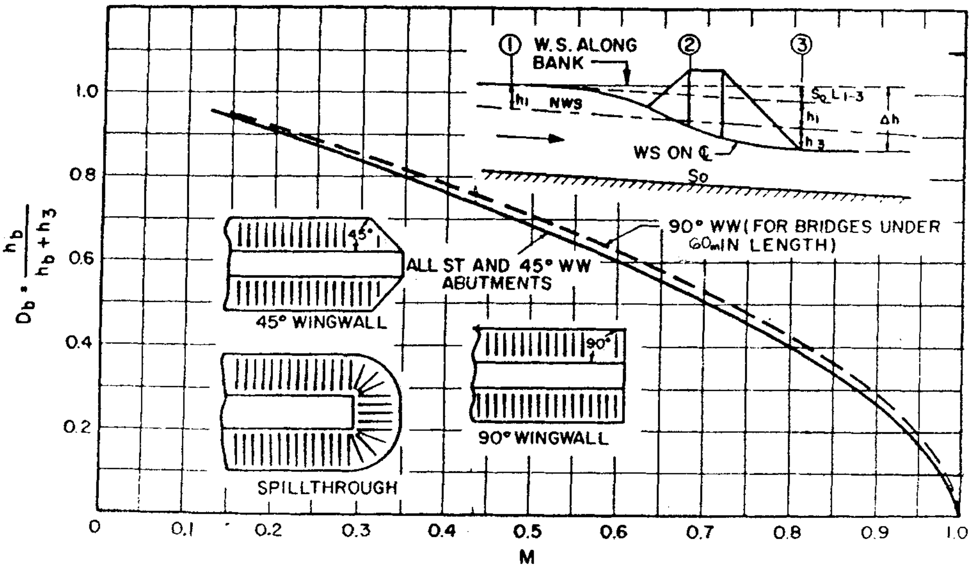

Fig. 13.8 Differential water level ratio base curves

The water level downstream of the constriction is h_3 below normal level. With piers, it was found that although backwater h_b increased, h_3 remained as for a constriction with no piers.

In the case of eccentric or skew crossing one adds the additional backwater Δh_c or h_s to h_b and determines h_3 from

$$h_3 = (h_b + \Delta h_c \text{ or } \Delta h_s)\left(\frac{1}{D_b} - 1\right) \qquad (13.14)$$

where D_b is obtained as before.

Now the total difference in water level across the embankment is

$$\Delta h = h_3 + h_1 + S_o L_{1-3} \qquad (13.15)$$

where h_1 is the total backwater allowing for piers or eccentricity, S is the bed slope and L is the distance from section 1 to 3.

DISTANCE TO MAXIMUM BACKWATER

The distance from the waterline on the upstream face of the embankment to the point of maximum backwater was evaluated and is presented in a tentative chart by the U.S. Department of Transportation (1978).

The distance is of the order of

$$L = 5b\ \Delta h/\bar{y} \tag{13.16}$$

where b is the width of opening at the waterline, Δh is the drop in water level across the opening and $\bar{y} = A_2/b$ (13.17)

Since Δh is a function of L it is necessary to estimate L first, and then calculate Δh and then revise L.

COMPLEX STRUCTURES

In the case of two bridges in close proximity the backwater effect is not necessarily the sum of the individual effects. The closer the two bridges, the nearer the resulting effect approaches that of one bridge. The backwater was found to increase by 30 to 50% for a distance between two identical bridges varying from three to ten times the embankment width at waterline level in the direction of flow. It would be wise to model the system in a hydraulics laboratory in order to confirm the water levels where complex bridge structures are contemplated.

Where scour of the bed is possible under the bridge (Laursen, 1962) the backwater effect may be reduced due to the reduced velocity through the constriction. Spur dykes (Fig. 13.9) have been found to assist greatly in reducing scour where it is likely to be a problem. Diving currents beside steep banks and around piers have been known to cause estensive damage to foundations.

OBSTRUCTION BY BRIDGE PIERS

Although the approach embankments are generally the major contraction effect on the channel width, piers across the section can also add to the backwater. The obstruction is aggravated by the contraction of flow between the piers.

The backwater effect of piers perpendicular to the flow was investigated in detail by Yarnell (1934) and Lin et al (1957). The parameters employed were the Froude number $F = v/\sqrt{gy}$ at the downstream section and the pier width to span ratio R. The research applies to subcritical flow although the depth could also pass through critical beyond the piers. The results may be summarized by the equation

$$\Delta y/y_3 = KF_3^2\ (K + 5F_3^2 - 0.6)\ (R + 15\ R^4) \tag{13.18}$$

where K is characterized by the pier shape in accordance with Table 13.1. The figures are for pier length to width ratio of four, and K reduces slightly for longer piers.

especially if air is trapped between girders. In this case the lateral force required to dislodge the bridge deck is reduced since frictional resistance is reduced.

Flow conditions under a submerged bridge are similar to those through a culvert with inlet control. Flow may also occur over the top of the embankment and bridge. This is a case of a broad crested weir.

The depth/discharge relationship over a broad crested weir such as an embankment is more difficult to analyze than for a sharp crest on account of the unknown position at which critical depth occurs, and the problem of evaluating energy losses. The hydraulics of flow over a broad crested weir may be studied using momentum principles and neglecting friction. Equating the net force on the water body between sections 1 and 2 in Fig. 13.10 to change in momentum,

$$wy_1^2/2 - wy_2^2/2 - wh(y_1 - h/2) = (wq/g)(q/y_2 - q/y_1) \qquad (13.19)$$

Solving for q, the flow per unit width of crest,

$$q = \sqrt{[y_1^2 - y_2^2 - h(2y_1 - h)]\, gy_1y_2/2(y_1 - y_2)} \qquad (13.20)$$

Chow (1959) indicates that experiments have proved that

$$y_2 \doteq (y_1 - h)/2 \qquad (13.21)$$

and that $q = 0.612\sqrt{g}\left[\dfrac{y_1}{y_1 + h}\right]^{\frac{1}{2}} (y_1 - h)^{\frac{3}{2}}$ (13.22)

or $q = C\sqrt{g}\,H^{\frac{3}{2}}$ (13.23)

where $H = y_1 - h$. Over the maximum range of h from zero to y_1, C could vary from 0.612 to 0.432. From observations it is found to vary from 0.54 for low sill height to 0.47 for a high sill or weir.

The previous theory applied to a weir or sill with the tailwater level above or below the critical depth over the weir. If the tailwater is lowered below the critical depth level, the depth over the sill will fall until it reaches critical depth. At this stage the specific energy of the flowing water, $y + v^2/2g$, is a minimum, and the critical depth is given by

$$y_c = \sqrt[3]{q^2/g} \qquad (13.24)$$

When the tailwater drops below the critical depth over the sill it no longer affects the flow conditions over the sill. Actually the critical depth for a free overflow occurs a little way upstream (about $3y_c$) from the crest. The depth at the crest is less than y_c on account of the non-parallel flow. Depth is found for a free drop crest to be $y_c/1.4$, so that the flow in terms of the depth over the crest y_o is

$$q = 1.65\sqrt{g}\,y_o^{3/2} \qquad (13.25)$$

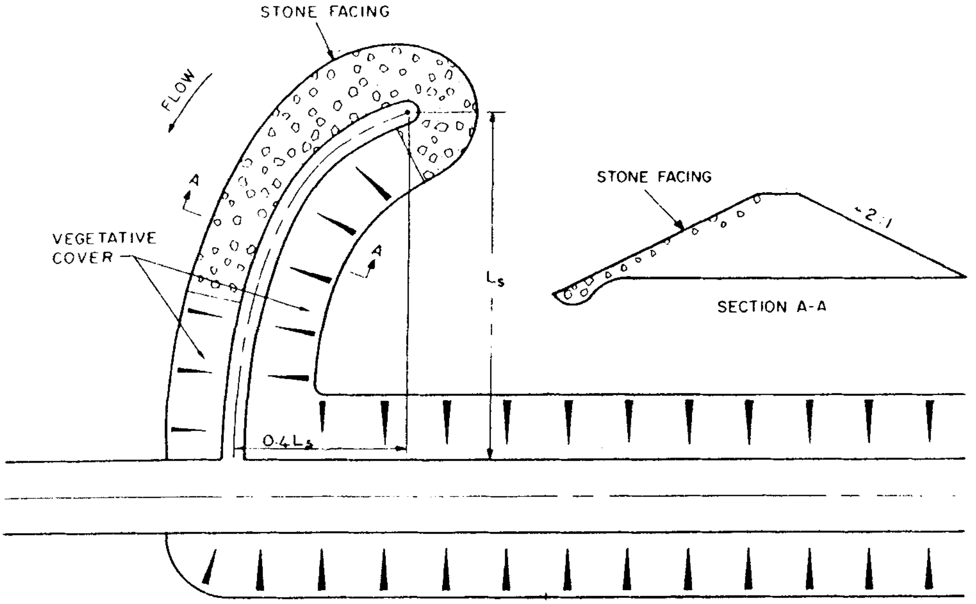

Fig. 13.9 Plan and cross section of spur dyke

TABLE 13.1 Pier Shape Factor K

	K
Semi-circular nose and tail	0.90
Lens-shaped nose and tail	0.90
Twin cylinder pier with connecting diaphragm	0.95
Twin cylinder pier without diaphragm	1.05
90° triangular nose and tail	1.05
Square nose and tail	1.25

INUNDATION OF BRIDGE

If upstream water level rises to above the soffit of the bridge, flow conditions may alter. If the water touches the upstream face, orifice flow may result instead of free flow. Discharge is then proportional to the square root of the head and not the head to the power of 3/2. In this case the upstream water level rises considerably in order to achieve an increase in discharge when compared with free surface discharge. Inundation of the roadway is highly likely. Other problems also arise if this type of flow occurs. Damage to the superstructure by floating objects is possible. The opening may become blocked more easily by floating debris. There may occur uplift under the superstructure.

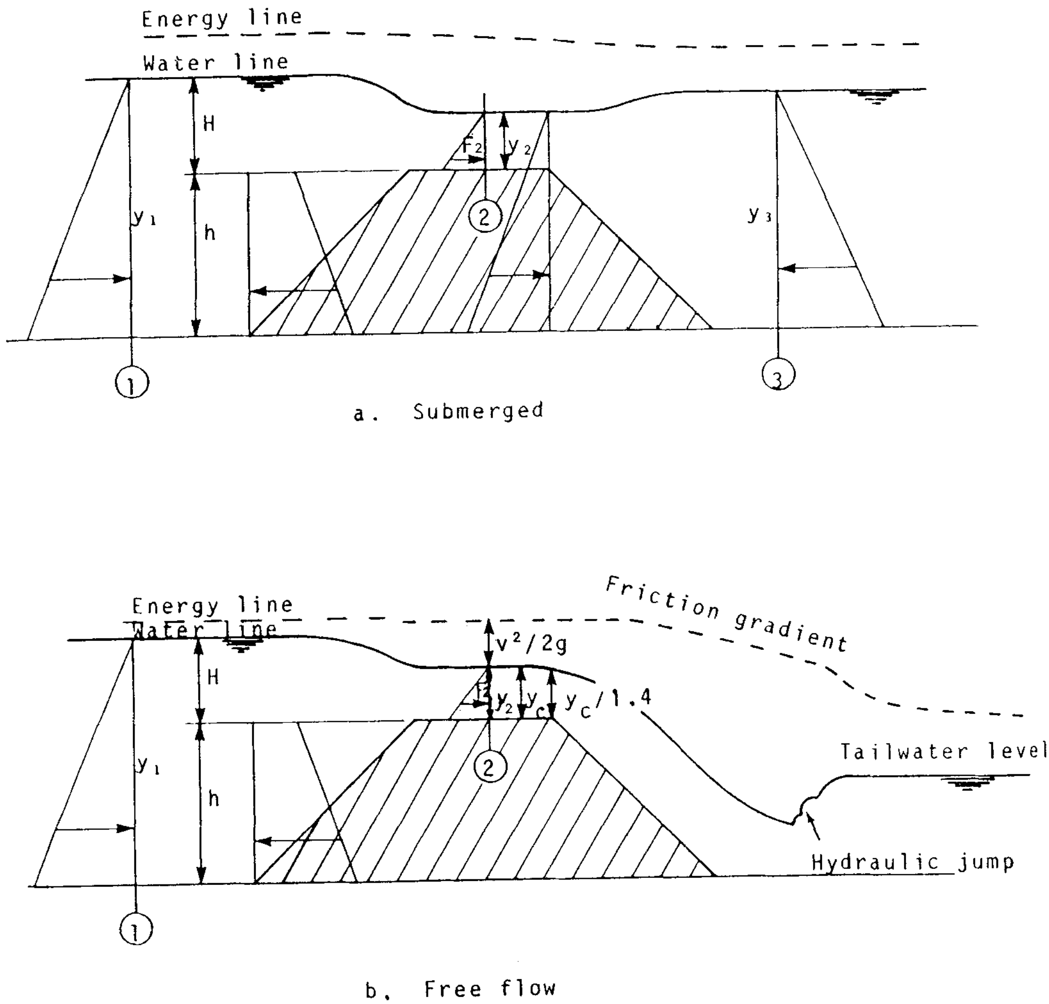

Fig. 13.10 Broad crested weir flow

EROSION DUE TO OVERFLOW

It is now possible to estimate the maximum height of embankment to avoid erosion by flow over the crest. The scour velocity over the crest may be estimated from the equation

$$d \approx \frac{0.25\ v^2}{g(S-1)\ \cos\ \Theta(\tan\phi\ -\ \tan\Theta)} \qquad (13.26)$$

where d is the erosive particle size.

For a flat horizontal crest, with ϕ equal to 35°, this gives the permissible velocity over the crest as

$$v_2 = 1.6\sqrt{dg(S-1)} \qquad (13.27)$$

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