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Stresses in the lithosphere caused by glacial loads

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Abstract

Simple elastic theory shows that horizontal stresses caused by a surface load which has a similar wavelength to the flexural wavelength of the lithosphere exceeds the vertical stress by several times. The enhancement factor is very sensitive to the wavelength. For glacial timescales, the wavelength of the British ice sheet was close to the flexural wavelength, while the Fennoscandian one was much larger. For models which allow time-dependent glacio-isostatic rebound, it is inferred that the stresses due to both the British and Fennoscandian ice loads were large until the end of glaciation, but very small by the present.

1 Two-layer elastic over fluid flat Earth model

For a uniform, incompressible elastic layer overlying an inviscid fluid halfspace, the deformation in response to surface loads of arbitrary wavelengths can be calculated analytically. To determine the response to more general loads in two dimensions or with axial symmetry, the load may be expressed as a sum of Fourier or Hankel components and the total response is the sum of responses to each component. The continuum version of Newton's second law is (eg [5, 8])

$$t_{ij,j}^{(\delta)} + (p_{,j}^{(0)} u_j)_{,i} - \rho^{(\Delta)} g_i^{(0)} - \rho^{(0)} g_i^{(\Delta)} = 0 \tag{1}$$

where t is the Cauchy stress tensor, p is the pressure $(= -t_{kk}/3)$, ρ the density and $\boldsymbol{g}$ the gravity. The superscripts (0), (δ) and (Δ) indicate the initial, material incremental and local incremental fields respectively. If perturbations of the gravitational field are ignored, and incompressibility assumed, the last two terms do not enter the equation.

Calculations have been made for the deformation and stress caused by a load of given wavelength for a 65 km thick elastic incompressible plate of density 3000 kg/m^3 and shear modulus 4×10^{10} Pa overlying an inviscid half-space. The elastic thickness is chosen to coincide with estimates of elastic thickness determined from postglacial rebound in the British Isles [4]. The flexural wavelength of the elastic layer in the thin-plate approximation is

$$\lambda_f = 2\pi(Eh^3/(12(1-\nu^2)\rho g))^{1/4} = 660 \text{ km} \tag{2}$$

where E is the Young's modulus, h is the thickness of the elastic plate and ν is Poisson's ratio for the elastic layer.

In Figure 1, the dimensionless horizontal stress is plotted versus wavelength of the load and depth within the elastic layer. The vertical stress for the same load is unity at the surface and attenuates with depth, more sharply for short wavelengths than long wavelengths. The maximum amplification of the horizontal stress occurs at wavelengths close to the flexural rigidity where the bending of the lithosphere is greatest.

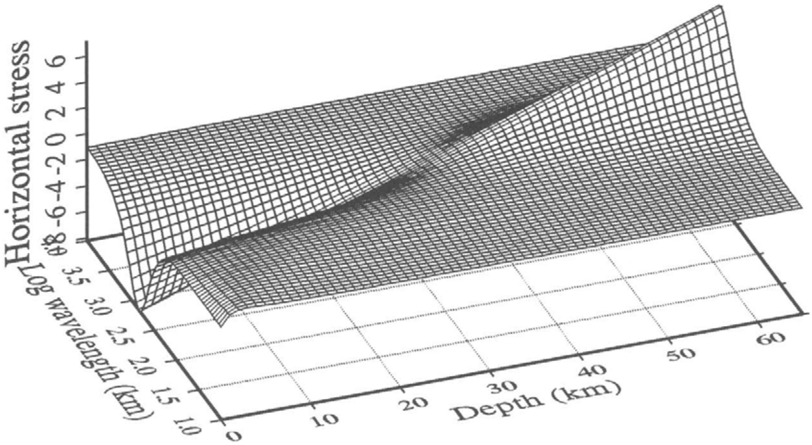

Figure 1: Horizontal and vertical stress as a function of wavelength and depth within the lithosphere normalised by the weight of the load. The maximum response factor for the horizontal stress is close to the flexural wavelength of the lithosphere.

1.1 Results for axisymmetric ice loads with elliptic profile

The deformation and stress field has been calculated for two ice sheet models which represent approximately the British ice sheet and Fennoscandian ice sheet at the last glacial maximum. Of particular interest is the maximum shear stress, which is equal to half the difference between the maximum and minimum principal components of stress. In this geometry, they will always be the radial horizontal stress and vertical stress. The vertical stress is constrained to be equal to the load at the surface and for ice loads of moderate to large lateral extent, there is little variation with depth within the lithosphere. The horizontal stress is much more dependent on the horizontal extent of the load compared with the flexural wavelength of the lithosphere. Because the smaller ice sheet (diameter 660 km) is much closer in lateral extent to the flexural wavelength, it produces larger stresses, despite being just over half the thickness. The maximum shear stress is related to the likelihood of seismicity and faulting occurring. If the shear stress is in excess of 10 MPa, then pre-existing faults may be re-activated [3]. Because the horizontal stress is much larger in magnitude than the vertical stress, it is the main contributor to the maximum shear stress. In Figure 2, we compare the radial stress predicted for two ice sheets of elliptic profile, one with radius 330 km to model the British ice sheet and the other with radius 1000 km for the Fennoscandian ice sheet.

2 Spherical Maxwell viscoelastic model

The deformation of the lithosphere has been calculated using the full equation (1) above, without any approximations, a spherical Earth model, and an elastic lithosphere overlying a Maxwell

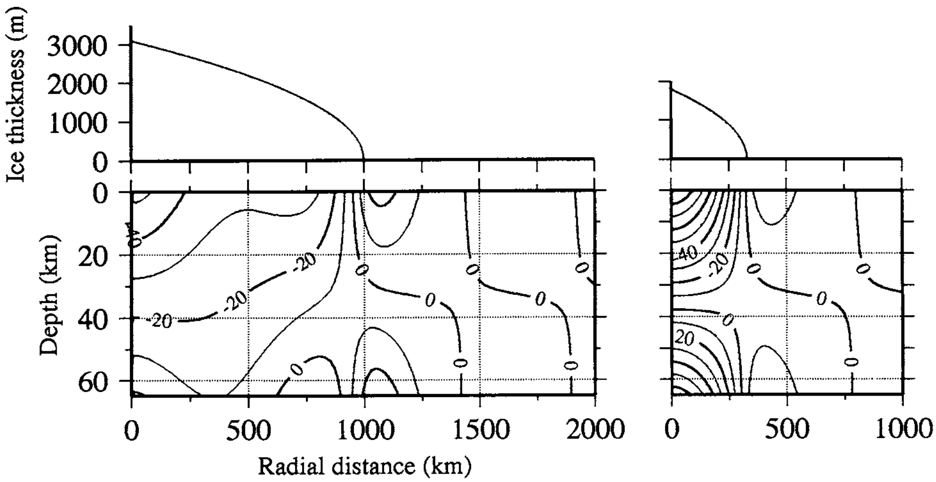

Figure 2: Horizontal radial stress (MPa) as a function of distance from the centre of the load and depth within the lithosphere for an axisymmetric ice load with elliptic semi-profile for two ice sheets of different lateral extent. The maximum compressional horizontal stress occurs at the centre of the ice sheet with a small amount of extension outside the edge of the ice sheet.

viscoelastic mantle, with seismically determined elastic properties [2] and mantle viscosities determined from fitting relative sea-level observations [4]. A glaciation/deglaciation cycle has been used to approximate the growth and decay of the Fennoscandian ice sheet. Figure 3 shows the maximum stress difference at the end of deglaciation and at the present. Because the Maxwell viscoelastic rheology behaves elastically on short timescales and viscously on long timescales, the effective flexural wavelength is time-dependent but with a lower bound of about 660 km as in the model above. Therefore, the maximum stress difference is somewhat smaller than in Figure 2. The values of stress are quite strongly dependent on the elastic properties of the various layers as seen by the sharp variation in stress at 15 and 25 km depth. Because most of the postglacial rebound is complete by the present, there remains little residual stress difference near the surface in the model. However, at the end of the glaciation, the stresses are still large enough to cause seismicity. This is consistent with observations of late glacial faulting [6] and the predominance of the NW-SE regional stress field in Fennoscandia [7] rather than a radial pattern which would be caused by postglacial rebound.

3 Conclusions

The maximum shear stress in the lithosphere has been calculated using the same models which fit relative sea-level observations. The calculations indicate that the stresses were large enough to cause faulting during and after the end of deglaciation, but the residual stress today is probably too small to be observed in comparison with the prevailing NW-SE pattern in Europe due to ridge push from the Mid-Atlantic ridge. The stresses at glacial maximum may have been larger for the British Isles than for Fennoscandia. The results of the modelling are consistent with

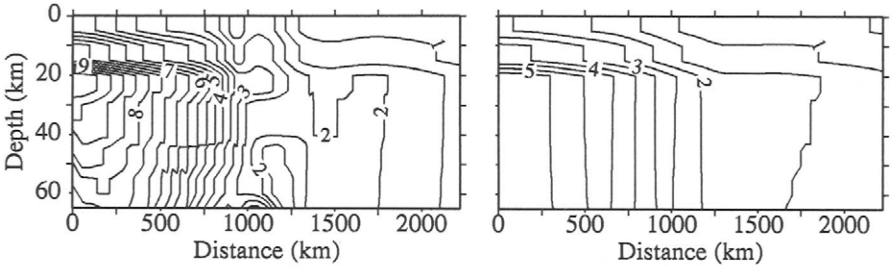

Figure 3: Maximum shear stress (MPa) for the Fennoscandian ice sheet at the end of deglaciation (left) and at the present (right) for an axisymmetric ice sheet with maximum radius of 1000 km at 18 thousand years before present (kaBP) and finished melting at 8 kaBP.

observations of late glacial faulting in both Fennoscandia [6] and Great Britain [1], and with the observed stress field and seismicity pattern in Scandinavia today [7].

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