ON METHODS OF OPTIMIZATION IN THE DESIGN OF WELL-FIELDS IN COASTAL AQUIFERS

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ABSTRACT

The purpose of this work is to present certain basic analytical procedures in connection with the problem of the optimum design of well-fields in coastal aquifers, these being under consideration at the present time in the Canary Islands. Some simple examples are given of designs of well systems in this type of aquifers, and conditions of efficiency of a technical and economic nature are discussed.

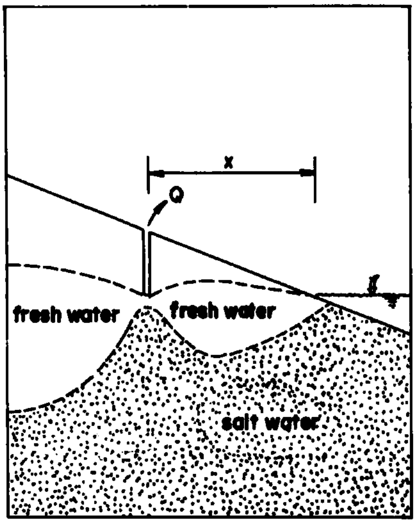
1. INTRODUCTION.

The hydrology of groundwater has undergone a noteworthy theoretical development in the last four or five decades. The same can be said of prospection and the techniques of capturing water resources in the subsoil; the progress made in the last decades both in well drilling and the construction of pumping equipment is evident. The process of technification of hydrogeological exploitation is not, however, devoid of its small gaps; for instance, current groundwater engineering has scarcely progressed beyond the design of individual extractions. The concept of hydrological unity is continually stressed, as well as the fact that aquifers are natural bodies in which the nature of unity and interrelationship among its parts are of prime importance, despite which formal models including such concepts are not generally available when promoting a specific mining operation.

2. THE PROBLEM OF THE DESIGN OF A WELL-FIELD IN A COASTAL AQUIFER.

The objective is to concretize the system of wells which will ensure the optimum exploitation of a coastal aquifer. The problem is essentially technical and hydrological but also involves an economic aspect. In general terms, the question lies in the search for a formula whereby the totality of the water extractions fulfils the rules of economization of the minimum total cost type and of equality in the margin between costs and profits.

The problem is studied by way of a very simple formula. Let us imagine, for the purpose in hand, a free and homogeneous coastal aquifer overlain by land with a sloping and totally regular surface. Let us assume, in addition, that the said aquifer is undefined in all directions from the semiplane defined by the (straight) line representing the coast, that the interface is a net surface that fulfils the law of Ghyben-Herzberg in all its extension and that the phenomena of salinization are induced exclusively by the ascent of the interface as a consequence of the lowering of the water table caused by pumping. Let us accept, moreover, that the capacity of transport and storage of the aquifer are not used to the best advantage, in such a way that the water is pumped at a uniform and constant rate and is used at points which are completely independent of the well sites. (This would be the case, for example, of water pumped to a canal for use in an area other than that where it was mined). In the absence of pumping and in a steady regime, the formula expressing the elevation of the water-table above sea-level would be:



Coastal aquifer: schematic section

$$h^2 = \frac{1}{K\beta}\left(p\,x\,(l-x) + 2\,q\,x\right)$$

where "p" represents the infiltration; "l", the depth of the aquifer perpendicularly to the coastline; "q", the amount of flow received by the aquifer at its inland extremity; "K", the permeability; "β", the quotient between the density of fresh water and the difference between those of salt and fresh water; and "x", the distance from the coastline to the point under consideration.

When water is withdrawn from the aquifer, the above formula becomes the following **(1)**:

$$h^2 = \frac{1}{K\beta}\left[p\,x\,(l-x) + 2\,q\,x - \frac{Q}{\pi}\,L\left(\frac{2x}{r}\right)\right]$$

("L" expresses neperian logarithms)

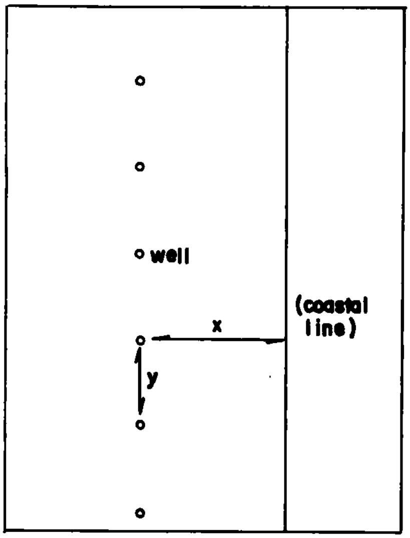
where "Q" represents the delivery pumped and "r" the radius of the well.

Let us now imagine an infinite line, parallel to the coast, of equidistant points of capture. Let "y" be the distance between any two wells and "Q" the uniform delivery pumped from each one. Under these conditions, the lowering of the water table at the vertical of each well will be given by the following equation:

$$h^2 = \frac{1}{K\beta}\left[p\,x\,(1-x) + 2\,q\,x - \frac{Q}{\pi}\,L\left(\frac{2x}{r}\right) - \left(\frac{Q}{\pi}\right)\Sigma L\left(1 + \frac{4\,x^2}{N^2 y^2}\right)\right]$$

("Σ" expresses the N-summation from 1 to infinity)

This expression has been formulated taking into account the existence of image wells due to the presence of a positive barrier with the coastline. Undoubtedly, this artifice is not strictly applicable to the case of free aquifers, but in our case it will be used in virtue of the small relative descents which will occur in the piezometric surface. On the other hand, the drops induced by pumping in the vicinity of the same extractions will be poorly represented by the formula in question, particularly because of the foreseeable slight penetration of the wells into the aquifer. The factor "L(2*x/r)" will thus be substituted by a "Do" so as to take into account the consequences of partial penetration. For the calculations involved in this work, the simplified Forchheimer formula has been used (2).



Line of wells: schematic plan

In view of the above, let us suppose that the drawdown tolerated in each of the wells is given by an f(x) type expression; whence, by calculating the summation in the preceding formula and considering the inverse of the distance between wells ("y") to be equal to the number of wells per unit length ("n"), it is found that:

$$Q = \pi\left[p\,x\,(1-x) + 2\,q\,x - K\beta\;f(x^2)\right] \Big/ \left[D_0 + L\,\frac{sh(2\pi\,x\,n)}{2\pi\,x\,n}\right]$$

("sh" expresses hyperbolic sine)

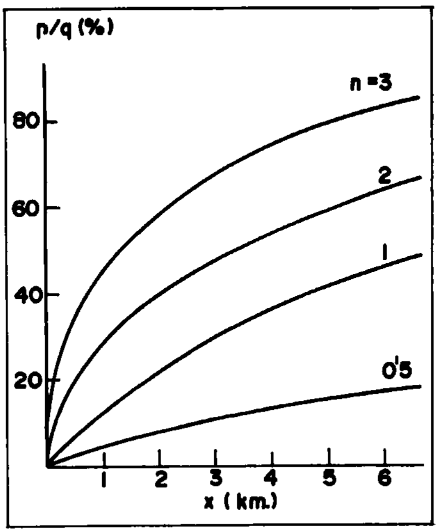
If both members of the above expression are divided by "y" it will be seen that:

$$\rho = \frac{\pi n \,(p\, x\,(1-x) + 2\, q\, x - K\beta f(x^2)}{D_0 + L\left[sh\,(2\pi x\, n)\ /\ 2\pi x\, n\right]}$$

where "ρ" represents the flow extracted from the aquifer per unit of coast length.

The above equation affords the volume which may be pumped as a function of the natural characteristics of the aquifer (recharges, permeability, etc.), of the construction conditions of the wells or of their hydrodynamic efficiency ("Do"), of their distance from the coast and of the separation between them. These last two variables are those the designer finds most at hand when planning a specific system of groundwater extraction in a coastal aquifer. The farther the wells are from the coast or the denser the line of extractions, the greater the delivery which can be put to use.

With regard to these two variables, the function "ρ" has the typical characteristics of what is known in Economics as the production function: it is monotonic increasing and displays decreasing marginal returns (derivatives). Several profiles of this function are shown in the adjoined Figure for a case which may be described as typical of the island of Tenerife: recharges in the order of $3Mm^3$/year/km and permeability of some 400 m/day. It has been supposed that the wells will not surpass sea-level and that the piezometric surface will be maintained 20 cm above sea-level (that is, the function "f(x)" will be constant and equal to 0.20 m.).



Productivity as a function of distance to the coast and nº of wells per unit length

The Figure shows that for a certain exploitation delivery there is an infinite number of possible combinations of separation between wells and distance to the coast. Naturally, of all of these, only one will be efficient from an economic point of view, that is, only one will afford the flow in question at a minimum cost. The problem is to determine which.

In order to find a solution, the costs of water extraction must be considered. In fact, the construction and exploitation of the wells whose water production is given by the above-mentioned formula will imply a cost ("C") per unit of coast length which can be entered in the accounts, for example, by means of a relationship of the following type:

$$C = n\,C_0 + n\,C_1 x + \rho C_2$$

"Co" represents the fixed cost associated with the opening of the water procurings, which cost will prove to be independent of well depths and of their discharge (for instance, the value of lands necessary for the pump-sites, the cost of access (by road), the cost of construction at the surface, the cost of laying-on power-lines and any other cost of a similar nature). "Co" will be expressed, in consequence, by means of a certain amount of pesetas per well.

"C_1" is the *coefficient for the costs which prove to be proportional to* the depth and number of the wells (and, specifically, the cost of drilling). "C_1" will therefore be obtained by multiplying the price of drilling the unit length of the well by the slope of the terrain (providing, of course, that this is approximately constant) and will be expressed in pesetas per metre.

"C_2" is the coefficient of the costs proportional to the delivery exploited. It will refer, in particular, to the cost of pumping the water and, in part, to the purchase of equipment. If "C_2" refers only to the costs of pumping, it can be obtained by way of the formula:

$$C_2 = \frac{2{,}722\ h\ Kw\ A}{R}$$

where "h" is the height (m.) to which the water is raised; "kw", the price per kilowatt-hour; "A", the factor of updating the current of expenditures involved in the power required to raise the water, which factor will prove to be a function of the discount rate and of the period of useful life attributed to the installations; and "R", the total power yield of these. The dimensions of "C_2" will consequently be of pesetas per unit delivery.

We have, thus, a function ("ρ") which gives the volume of water it is technically feasible to extract from the aquifer, and a further function ("C") which gives the costs of the extraction in question. It will therefore be possible to find those values of "x" and "n" which maximize flow for a given cost or which minimize costs for a precise delivery pumped. Such values will be obtained by determining the maximun of the former function subject to the restriction imposed by the existence of the latter. By applying the rules of conditioned maxima it is found that:

$$(\partial \rho/\partial x)/(\partial C/\partial x) = (\partial \rho/\partial_n)/(\partial C/\partial_n)$$

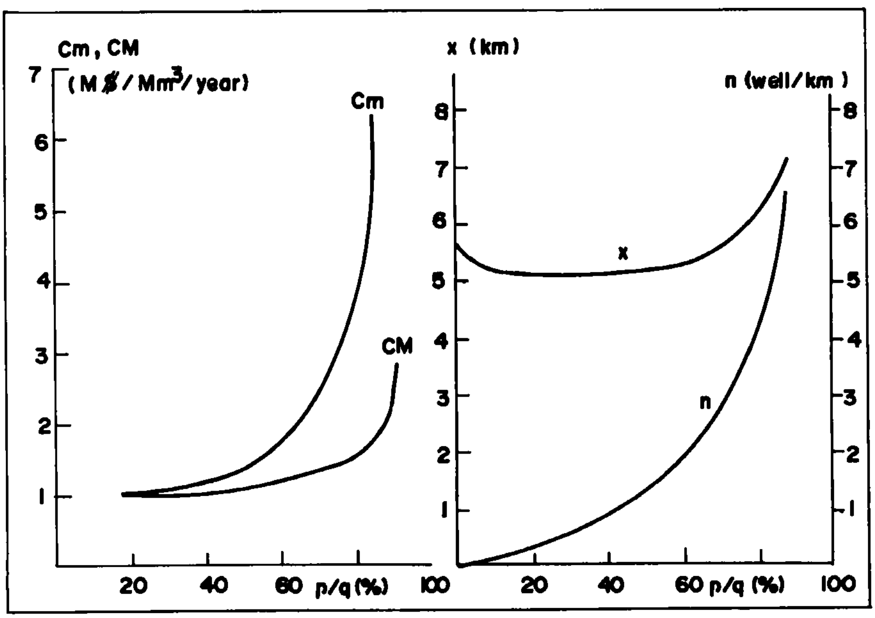
which later becomes as follows:

$$x = C_0(\partial \rho/\partial x)/\left[n(\partial \rho/\partial_n) - x(\partial \rho/\partial x)\right]$$

By way of an example, an analysis of this type has been prepared by considering a function of costs having the following coefficients: C_0= 40,000 U.S. $ per well; C_1= 64,000 U.S. $ per kilometre; C_2= 800,000 U.S. $ per Mm^3/year. (1 U.S. $ = 125 ₧).

These coefficients are normal with regard to the costs and procedures involved in water procuring in coastal aquifers in some areas of the island of Tenerife. In order to obtain them, we have considered that the coefficient of updating ("A") equals 10 (which amount is, approximately, the current value of a monetary unit in use over a period of 50 years and with a discount rate of 10%). As for the others, the hypotheses under which the curves of the previous graph were obtained have been used.

The "CM" and "Cm" curves of the following diagram represent, respectively, the average and marginal costs of the extraction of the water resources from the hypothetical aquifer under consideration, which curves are obtained by resolving the equation shown above. The optimum exploitation from the

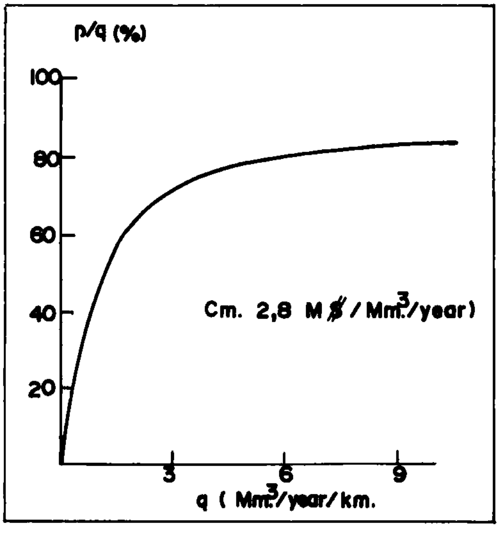


Average and marginal exploitation costs

economic point of view will, of course, be achieved at the point where costs and marginal profits are equal and, should there exist a market price for water resources (as it usually occurs in the Canaries), at the point where the market price of the water and the marginal cost of extraction are equal. (It should be stressed, however, that the price of water referred to in this article is not the immediate price that may occur in that market but the current value associated with the current of same extended throughout the period of useful life of the exploitation).

In any case, and in the eventuality that the objective of the exploitation were the compulsory procuring of a certain delivery, the "Cm" curve would by itself afford information on the marginal cost involved in such operation.

Maintaining the above suppositions, and for the current prices of water in the island of Tenerife (around an average of 0.28 $ per cubic metre at present), the optimum exploitation of the aquifer would lead us to abstract little more than 70% of its base delivery. Logically, and as the following graph displays, this percentage proves to be a function of this base delivery. In the said graph, the fraction of the flow of the aquifer is recorded which it would be possible to capture at a price of water of 0.28 $/m^3 (that is, as explained above, for a marginal cost of 2.8 M$) as a function of its resources and without modifying the remaining hypotheses of the example.



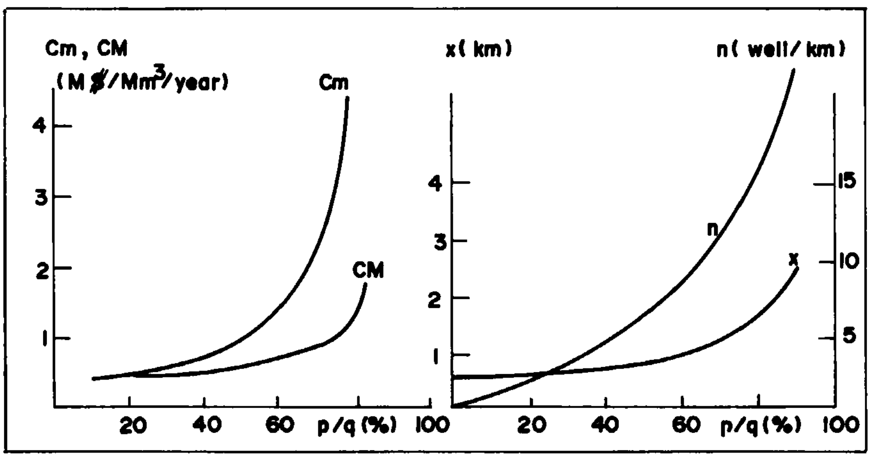
An example with actual figures (Tenerife)

It must be borne in mind, in any case, that, under the suppositions presented in this work, the most economical mining will be verified effectively by means of a line of wells such as that used to develop our reasoning. This is easy to intuit and not very difficult to demonstrate.

The analysis presented is no more than an elementary example for the purpose of illustrating the technique of the analytical procedure, the application of which can be extended to different cases. Meanwhile, in effect, a function of costs different to that proposed above can be conceived. Let it be of the following type:

$$C = n\,C_0 + n\,C_1\,x + q\,C_2\,x$$

In this case, the cost of pumping does not depend only on the discharge raised but also on the depth of the well, which depth will determine the height to which the water is lifted. Repeating the hypothesis of an infinite line of water collections, maintaining all the suppositions relative to the characteristics of the aquifer of the earlier example, and with values of the coefficients of costs such as these: C_0= 40,000 \$ per well; C_1= 64,000 \$ per km; C_2= 160,000 \$ per Mm^3/year and Km, the cost curves of the following diagram will be obtained.



Average and marginal exploitation with consideration of well depth

The position of the line of wells will now be different: much closer to the coast, since it will prove preferable to save pumping power even at the cost of rendering the line of wells denser in extractions. The curves do not present any further noteworthy features.

A slightly more complicated case would be that of the exploitation of the coastal aquifer referred to previously, by means of a line of wells of a finite length. Imagine, for instance, that, even if the aquifer were to extend indefinitely, the wells could only be drilled in a certain strip of terrain, which is limited by two lines perpendicular to the coast line. (The problem could appear, in practice, in a case where the aquifer is economically exploitable only from a valley bordered on either side by mountain systems, as occurs occasionally in the Canaries). In addition, let us suppose that the function of costs coincides with the second of those presented earlier in this work.

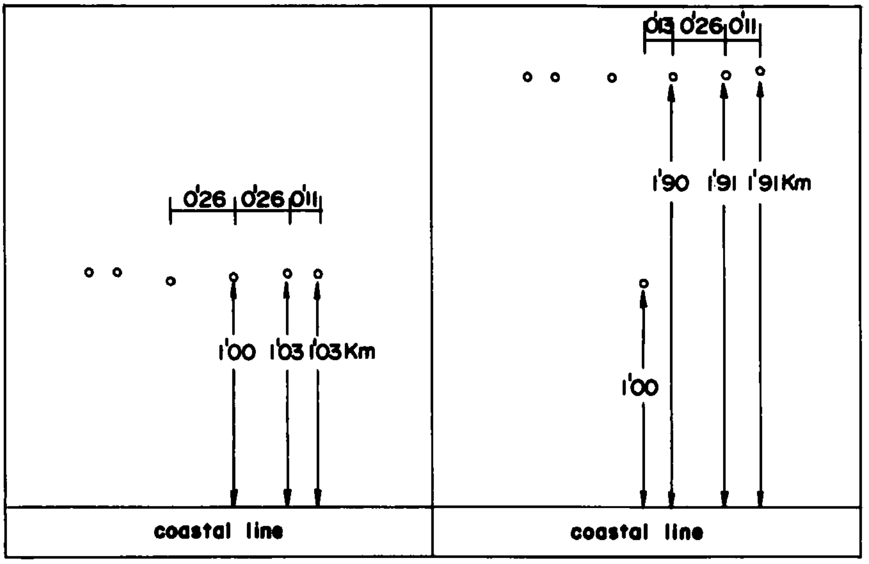
As expected, under these hypotheses, the optimum set of points of capture will not be based on a line of uniformly separated wells. The wells at the extremes will have to be situated farther from the coast than those in the centre, and the distance between the latter wells will be, in general, greater than that between the former ones.

The problem can be solved on the basis of establishing the flow withdrawn from the totality of the wells as a function of the position coordinates of each well. The issue is then to determine the consequent equations with calculating the maximum of this delivery conditioned by the cost function. The affair is certainly a very boring exercise in differential analysis, the solution to which can only be obtained by means of computers.

The graph in the following page provides two solutions to the problem in hand, referred to cases in which six and seven wells have been allocated to cover a strip of land of a supposed width of one kilometre.

Certainly, the advantages of situating the water collections according to these canonical dispositions are not very relevant, especially for low densities of wells. For example -and adhering at all times to the suppositions of our exercise-, the cost of obtaining the same discharge that could be obtained on the basis of the seven wells in the earlier diagram would only be 6% lower than that of a line of the same number of wells, but uniformly distributed.

It will be readily understood that the schemes presented are no more than a simple analytical framework, illustrated by a succession of more or less fortunate examples. The method can, indeed, be readily extended to different cases. The problems of calculus may not be simple, but seldom so difficult that they cannot be undertaken with a modest PC. In any case, a production function of water will be obtained; again, a cost function should be available which, of course, need not coincide with those presented in



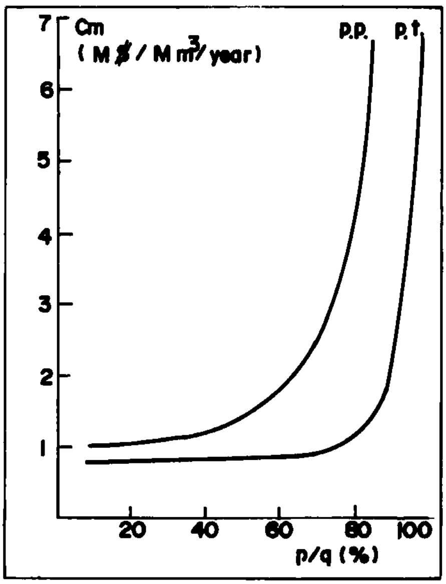
Two solutions for limited areas

this work. In every case, however the optimization of the system will lead to a generally increasing marginal costs law which, in some cases, will be notably increasing starting from a certain intensity of exploitation of the aquifer resources.

The formulation of these laws may prove to be complicated but should present few theoretical problems. However, sound criteria in regard to a series of issues of relative importance will be significantly required.

First, an accurate estimate will be needed of the "levels of safeguard" (that is, which should be the minimum level of the water in the wells above that of the sea) in order to guarantee the quality of the volumes abstracted. This will imply the best possible knowledge of the aquifer regime, particularly of the thickness of the transition zone between fresh and salt water and, in general, of the behaviour it displays in pumping processes.

Second, there is the issue of optimizing the penetration of the wells in the aquifer, a point of prime importance. On the basis of exclusively technical reasons there will be an optimum penetration, since if the wells penetrate only slightly it is more likely that the contamination induced by the ascent of the more salinized levels of the aquifer will be avoided, considering that the zone of filtration will be far away from the said levels; however, at the same time, this will bring about a greater lowering of the water table, thus facilitating contamination by ascent. The opposite will take place if the wells are deeper; in this case, the salinized levels will more readily reach the water procurings, but their ascent will be counteracted by the lesser drawdowns induced in the said levels. There is only one, basically empirical, solution to the problem, since the behaviour of the zone where the fresh and salt waters mix cannot be treated analytically, although, on a purely theoretical level, there is no reason why the penetration of the wells in the aquifer cannot be considered to be a further variable in design. In any case, this penetration, as well as the general efficiency of the captures, determine to a great extent the solution to the problem of the dimension of the wellfield and the costs of its exploitation. In effect, the less that efficiency, the greater the number of wells which will have to be constructed in order to extract the same delivery. In the graph below the curves of marginal and average costs contributed in the preceding pages are presented in comparison with those which would have been obtained in the case of total penetration. (In all the cases employed in this work, the efficiency of the wells was considered to be 100%). The great differences caused by this circumstance can be observed. In the Canary Islands, wells are generally only slightly penetrative, because of the high permeability of the coastal aquifers. They are constructed in accordance with traditional techniques, well adapted to the conditions of the groundwaters, by means of large-diameter vertical perforations (some three metres) to which bottom horizontal galleries are added for the purpose of facilitating infiltration and improving efficiency of water collection. Actually, water authorities usually prohibit drilling under sea-level.



Partial and total penetration: marginal cost curves

Reference will now be made, to a particular aspect of the economic dimensioning of well-fields. Even when the curve of marginal costs of a project is available and despite the fact that the system of prices is perfect and totally transparent, -which prices will provide accurate information on the marginal profits obtained from exploiting the aquifer-, it is clear that this will not be sufficient to throw light definitively on all the doubtful points.

The lack of water resources is a worldwide problem. If any water market exists, prices usually suffer from very appreciable processes of relative increase. Therefore, if we choose the present price of water to dimension the well-field, this latter is very likely to be eventually under dimensioned, that is, that the discharge of water collected will prove to be less than the optimum flow which would have been obtained if the escalating prices or increasing scarcity had been taken into account. The issue is, therefore, to formulate a forecast of what will be the growth-path of such prices or scarcities.

3. SUMARY AND CONCLUSIONS.

1º The design of a well-system for the exploitation of a coastal aquifer is not exclusively a technical issue. On the contrary, it presents a fundamental economic aspect. Its analysis requires, in principle, the selection of the variables relevant to the dimensioning of the well-field, such variables being, for example, the distance of the wells to the coastline, the number of wells and the distance between them, the penetration of the zone of filtration and so forth.

2º A large number of combinations of water captures can be applied to the exploitation of a coastal aquifer, but only certain ones will prove to t efficient from the technical and economic points of view. The determination of these combinations will lead to the establishment of water production functions and, hence, to functions (of marginal costs) which will include a relationship between the volume or flow of water produced and the marginal cost of producing it. The flow to be extracted will be deduced from the amount of costs that the pumpers are willing to bear, and from this amount the definitive value of the variables included in the analysis will be deduced.

3º Analyses of this type, corresponding to a short series of simple cases generally related to free and infinite coastal aquifers, are presented in this work.

4º In the cases presented, the circunstance arises (quite common, on the other hand) that the capture of up to 50% of the resources of the aquifer does not involve very high marginal costs. However, from this percentage upwards, the growth of these marginal costs of exploitation is quite noteworthy. In general, the extraction of 75% of the resources of the aquifer implies marginal costs which are already several times greater than those of capturing only a half of those resources.

5º In cases similar to those presented here, and for a certain marginal cost of obtaining the water, the fraction of the resources of the aquifer put to use will be larger, the greater the resources of the aquifer.

6º The analyses included in this work can be extended to the different cases which may be encountered in practice; in this regard, several problems will be presented here: one concerns the determination of the marginal cost functions pertaining to the case, which may prove to be complicated when conditions such as the geometry of the aquifer are not straight forward. Another more important problem is the estimation of the value of the physical factors conditioning the determination of the theoretical functions of production or of marginal costs: for example, the optimum penetration of the wells; the residual head of water which must remain in the piezometric level of the wells in order to ensure the quality of the water extractions, etc. Finally, there is the problem of establishing the economic variables of the analysis: what level of costs can be borne in the extraction of the resources of the aquifer; how can the costs of the analysis be related to the frequent price increases in the water market which tend to occur in so many coastal areas of the world, etc.

REFERENCES

(1) International Institute for Land Reclamation and Improvement, 1.972. Groundwater Hydraulics of extensive aquifers. ILRI Wageningen, The Netherlands, bulletin 13., p. 158.

(2) E. Custodio / M.R. Llamas, 1.976. Hidrología Subterránea, Ed. Omega S.A., Barcelona, p. 747.