THE BUFFER ROLE OF GROUNDWATER WHEN SUPPLY OF SURFACE WATER IS UNCERTAIN

Y. TSUR
Department of Economics and The Jacob Blaustein Institute for Desert Research, Ben-Gurion University of the Negev, Sede Boqer Campus, 84993 Israel

A. ISSAR
Water Resources Unit, The Jacob Blaustein Institute for Desert Research, Ben-Gurion University of the Negev, Sede Boqer Campus, 84993 Israel

ABSTRACT

When supply of surface water is uncertain, groundwater may serve as a buffer that mitigates undesired effects of this uncertainty. This paper explores the economic value of this buffer role of groundwater.

1 INTRODUCTION

Uncertain factors greatly affect many of our economic decisions regarding utilization of the environment. Water resources are no exception (see for example, Szidarovszky et al., 1976; Taylor and North, 1976; Goicoechea et al., 1982; Mercer and Morgan, 1978). Uncertainty is especially eminent in situations where consequences of present decisions depend on future events, such as in dynamic planning problems. Resource allocation over time is an example. This paper deals with the role of a specific and commonly found type of uncertainty in the development and use of groundwater: uncertainty in the supply of surface water.

We envisage a situation in which an existing source of surface water is augmented with a groundwater source. In the absence of uncertainty in the supply of surface water, the only benefit incurred by the groundwater is due to the increased supply of water. If, as is commonly found in reality, the supply of surface water is uncertain, then groundwater plays a role in addition to that of increased water supply: the role of a buffer that mitigates the undesired effects of uncertainty in the supply of surface water. In this paper we give a concrete content to the value of this buffer role and evaluate it explicitly in a particular context. We find that it is not unlikely that the buffer value of groundwater is of the same magnitude as the benefits due to the increased water supply.

To place our analysis in the context of a management of a stock of groundwater, we outline, in the next section, a simple dynamic management model under certainty. The following section introduces the concept of buffer value and provides an explicit expression for this value in a particular setting. Section 4 discusses implications of the buffer value when combined with the dynamic framework of Section 2. We conclude the paper with a brief summary.

2 A MANAGEMENT MODEL UNDER CERTAINTY

A result of the profit maximization excercises of firms (which are potential users of the water) is a relation which attaches to any level of water price a quantity to be demanded at that price. The inverse of this function is the derived demand for water $D(R_t, Q_t; a_t)$. For each time t, R_t is the quantity of water demanded, Q_t is the water quality, and a_t is a vector representing technological and economic parameters. The function $D(\cdot)$ is assumed to decrease in R_t and to increase in Q_t. Since we are concerned with groundwater, the quality index Q represents the level of purity in terms of salinity. We further note that this measure depends mainly on the volume of the remaining reserves of the aquifer, S_t. Thus $Q_t = Q(S_t)$ is a hydrological relation that translates stock levels S_t into quality levels.

The marginal extraction cost, $MC(R_t, S_t; b_t)$, is the cost of pumping one additional m^3 of water. At each time t, it depends on the extraction level R_t, the remaining reserves S_t, and on a parameter vector b_t representing the cost of the extraction technology and energy prices. S_t is inversely related to the depth of the water table, hence MC is expected to decrease in S_t. Usually MC is non-decreasing in R_t.

The aquifer's stock at time t changes as a result of extraction R_t and of recharge G:

$$\dot{S}_t = G - R_t \tag{2.1}$$

where $\dot{S}_t$ is the rate of change (time derivative) of S_t, and G is the natural rate of water recharge. A feasible extraction program R_t cannot use more than is available. Thus at each time instant t, R_t must satisfy

$$0 \leq R_t \leq S_t + G. \tag{2.2}$$

A given aquifer is thus characterized by the initial stock S_o, by the rate of water recharge G, by the marginal cost function (as related to the depth of water table and its response to changes in S_t), and by the hydrological relation Q(S) translating stock levels S into water quality measures Q.

Let $W(R_t, S_t)$ be the surplus, both to consumers (users of the water) and producers (sellers of the water), resulting from pumping R_t m^3 of water when the aquifer's stock is at the level S_t:

$$W(R_t,S_t;a_t,b_t) = \int_0^{R_t} [D(v,Q(S_t);a_t) - MC(v,S_t;b_t)]dv. \tag{2.3}$$

Define the welfare criterion function to be the discounted sum of producer plus consumer surpluses

$$J = \int_0^T e^{-rt}W(R_t,S_t;a_t,b_t)dt \tag{2.4}$$

where T is the planning horizon and r is the time rate of discount. The management problem is that of choosing the extraction policy R_t, $0 \leq t \leq T$, in order to maximize J subject to (2.1)-(2.2). In cases where the planning horizon T is not specified a-priori it can be augemented as an argument to the maximization problem.

Solving such a resource allocation problem has become a common practice (see e.g., Clark (1976) and Dasgupta and Heal (1979), among many others) and we shall not dwell on this issue here. Let R_t^* be the optimal trajectory of groundwater extraction (assuming such a solution exists) and let J^* be the corresponding level of the objective function. Then the development of the aquifer is justified on economic grounds if the benefit J^* exceeds the fixed cost investment needed to make utilization of the aquifer possible. There may be other criteria that guide policy makers in their decision on whether to develop a given groundwater stock, but these are beyond the scope of this paper.

3 THE BUFFER VALUE OF GROUNDWATER

When the model set forth above is applied in practice, it is most likely to encounter various types of imperfect information on some or all of its components. Some of this lack of information is inescapable either because acquiring the information is too costly or because it is affected by exogenous uncertain factors (weather). The randomness embodied in the supply of surface water is an example of such unavoidable uncertainty.

In this section we attempt to contribute to the understanding of a very specific role of groundwater, namely, that of a buffer that mitigates effects of uncertainty in the supply of surface water. The situation perceived is of an existing source of surface water which supplies a stochastic amount of water every year. A groundwater source then becomes available and causes: (i) an increase in the quantity of available water; and (ii) a change in the randomness of the supply of water. It is the monetary value of the neglected second effect that we try to measure.

The quantity of surface water available during a given year, X, is stochastic. Thus, at the beginning of the time period, X is regarded as a random variable with mean m and variance s^2. Water is being used as an input to a single output production process represented by the production function $F(R)$, where R represents quantity of used water. It is assumed that F increases with R in a diminishing rate: $F' > 0$ and $F'' < 0$, where F' and F'' are the first and second derivatives, respectively. Output price P is assumed to remain constant and the surface water is assumed to be provided without cost.

The derived demand for water function $D(R)$, shown in Fig. 1, is the result of maximizing over R the profit function $P \cdot F(R) - wR$ for various levels of water price w. Any quantity realization, x, of the surface water yields the profit $PF(x)$ which is merely the area beneath the derived demand function $D(R)$ to the left of x. Specifically, the mean m yields a profit level given by the area(ABmO) of Fig. 1. In the absence of groundwater, the producer simply takes whatever surface water is available and enjoys the random profit $PF(X)$.

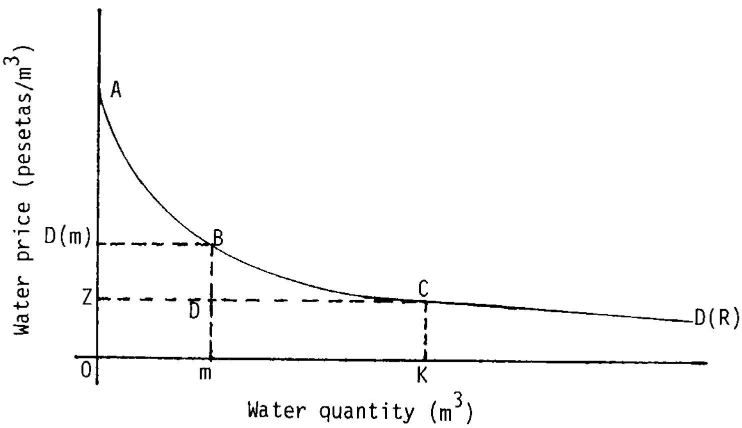

Fig. 1. D(R) is the derived demand for water; marginal extraction of groundwater is fixed at the level Z; the area(bcd) is the groundwater benefit incurred by the average increases in the supply of groundwater

Suppose that, at the beginning of the period before the realization of X is known, the producer (i.e., the potential consumer of the groundwater)is asked to specify the minimum certain income he would be willing to receive instead of the random profit $PF(X)$. The answer is denoted as the certainty equivalent income associated with the random profit $PF(X)$. Assume that the producer seeks to maximize expected utility of profit and, for simplicity, assume further that he is risk neutral. In this case the certainty equivalent income is simply the expectation $E(PF(X))$ which, using Taylor series expansion, can be approximated by

$$M_1 = PF(m) - 0.5P(-F''(m))s^2 \qquad (3.1)$$

The first term on the right hand side of (3.1) is the area(ABmO) in Fig. 1. The second term will be identified shortly as the benefit from groundwater due to its role as a buffer to the uncertainty in the supply of surface water. Thus we define $BV = .5P(-F''(m))s^2$, or equivalently

$$BV = 0.5 \cdot D(m) \cdot A(m) s^2 \tag{3.2}$$

where $D(m) = PF'(m)$ is the value of the marginal productivity of water evaluated at m, and $A(m) = -F''(m)/F'(m)$ is a measure of the concavity of F at m. This presentation illuminates the components that determine the buffer value of groundwater.

Suppose now that a groundwater source (of practically unlimited amount) becomes available at a constant cost of z pesetas per m^3. From Fig. 1, it is clear that the amount of water to be consumed in the new situation is K, where the first X m^3 are taken from the surface source and the remaining K-X m^3 from the ground. It is assumed that K is greater than the maximum value that X can take. The producer now enjoys a random profit of $PF(K) - z(K-X)$ which yields the expected profit

$$M_2 = PF(K) - z(K-m). \tag{3.3}$$

In Fig. 1, PF(K) is the area(ACKO) and z(K-m) equals the area(DCKm). Hence

$$W = M_2 - M_1 = \int_m^K [D(R)-z]dR + BV = \text{area(BCD)} + BV. \tag{3.4}$$

The quantity W is the maximum sum of money the producer would be willing to pay, at the beginning of the year before the realization of X is known, in order to make the groundwater source available. It is the (annual) ex-ante monetary measure of the welfare gains incurred by the groundwater. As revealed by (3.4), this welfare measure is comprised of two parts. The first is the area(BCD) which is the incremental mean profit resulting from the additional K-X m^3 supplied (on average) by the groundwater source. The second part is the buffer value BV. It is a result of the uncertainty being shifted from production to cost.

The buffer value can be interpreted as follows. The production function F is concave in water input. Without groundwater, water input is simply X. Hence a downward deviation of X from m decreases F more than an upward deviation (of the same magnitude) would increase F. The (risk neutral) producer prefers stability at m over the random X in this case, and would be willing to pay some positive amount to achieve this stability. In the new situation, with groundwater available, production is stable at the level F(K). Thus the move from the initial situation (without groundwater) to the new situation (with groundwater) is the same as: first stabilizing X at the level m, and then providing an additional K-m m^3. BV is the increase in ex-ante welfare caused by the stabilization. The uncertainty involved in the randomness of X was not eliminated but rather, shifted to the cost component and there, due to the linearity in X, it is harmless.

Example

Suppose $F(R) = a - b/R$ if $R \geq b/a$ and 0 otherwise, where a,b are two positive constants. The inverse demand for water is given by $D(R) = Pb/R^2$ for $R \geq b/a$. Marginal extraction cost is constant at the level z. The first part of W (corresponding to area(BCD) in Fig. 1) is due to the increased supply of water as a result of the groundwater becoming available and is given by $\int_m^K (Pb/R^2 - z)dR$; where K is determined from the relation $Pb/K^2 = z$. Calculating this integral, taking account of the relation between K and z, yields the value $(Pb/m)[(k-m)^2/K^2]$.

Now $D(m) = Pb/m^2$ and $A(m):=-F''(m)/F'(m) = 2/m$. Hence $BV = 0.5D(m)A(m)s^2 = (Pb/m)[s^2/m^2]$. If $(K-m)^2/K^2$ is of the same order of magnitude as s^2/m^2 (dependent mainly on s^2, the variance of X), then the buffer value of the groundwater is of the same order of magnitude as the direct gains associated with the increase in the water supply.

4 IMPLICATIONS

Returning to the dynamic model of Section 2, the decision problem was that of choosing the optimal management scheme of a given groundwater reservoir, calculating the optimal benefit, and comparing the benefit with the initial cost of development. To avoid technical complications associated with the dynamic optimization task, we assume the economy is as described in the example of the previous section and that this situation repeats itself identically every year. We further assume that the natural rate of recharge into the aquifer exceeds, on average, the annual demand for the groundwater (i.e., $G > k-m$) so that there is no scarcity effect.

In this situation it is obvious that the optimal groundwater extraction policy would be: at each time t, wait until the realization x_t of X_t is known and then supply $K-x_t$ m^3 of groundwater (provided $K > x_t$). The uncertainty does not effect the management policy. However, to decide on whether to invest in development, the total benefit due to the groundwater must be known in advance. Ex-ante this benefit is random when the annual quantity of surface water available, X_t, is random.

Suppose first that the supply of surface water is non-stochastic and fixed at the level m, then the present value of the (infinite) stream of benefits associated with developing the aquifer is due only to the increase in the water supply and is given by

$$W^c = \frac{1}{r}(Pb(K - m)^2/(m \cdot K^2)) \tag{4.1}$$

(cf. the example in Section 3). In a case of constant supply of surface water, W^c is the appropriate measure of benefit that enters the cost/benefit analysis on whether to develop the groundwater source.

Consider now the situation in which the supply of surface water is stochastic with mean m and variance s^2. According to the example above, the appropriate measure of benefit in this case is

$$W^u = W^c + \frac{1}{r}(Pbs^2/m^3) \tag{4.2}$$

where the additional term is the present value of the (infinite) stream of buffer values. As demonstrated in the example, this additional term may be of a similar magnitude to that of W^c.

5 CONCLUDING REMARKS

The presence of uncertainty in the supply of surface water increases the value of a potential groundwater source as a result of the buffer role of groundwater. This statement seems to agree with basic "intuition". In this paper, we gave a concrete meaning to this "intuition". We provided an explicit expression for the buffer value of groundwater in terms of three components: the value of marginal productivity of water, the concavity of the production function (which uses water as an input), and the variability of the supply of surface water. We demonstrated that, when the variability of the supply of surface water is not negligible, the buffer value of groundwater may be of the same magnitude as the gains resulting from the increased water supply.

6 REFERENCES

Clark, C.W., 1976. Mathematical Bioeconomics: The Optimal Management of Renewable Resources. Wiley, New York, Chap. 2.

Dasgupta, P.S. and Heal, G.M., 1979. Economic Theory and Exhaustible Resources. Cambridge University Press, Chaps. 5,6,7.

Goicoechea, A., Krouse, M.R. and Antle, L.G., 1982. An approach to risk and uncertainty in benefit-cost analysis of water resources projects. Water Resources Research, 18: 791-799.

Mercer, L.J. and Morgan, W.D., 1978. Measurement of Economic uncertainty in public water resource development: An extension. Amer. J. Agric. Econ., 60: 241-244.

Szidarovszky, F., Bogardi, I., Duckstein, L. and Davis, D., 1976. Economic uncertainties in water resources project design. Water Resources Research, 12: 573-580.

Taylor, B.W. and North, R.M., 1976. The measurement of economic uncertainty in public water resource development. Amer. J. Agric. Econ., 58: 636-643